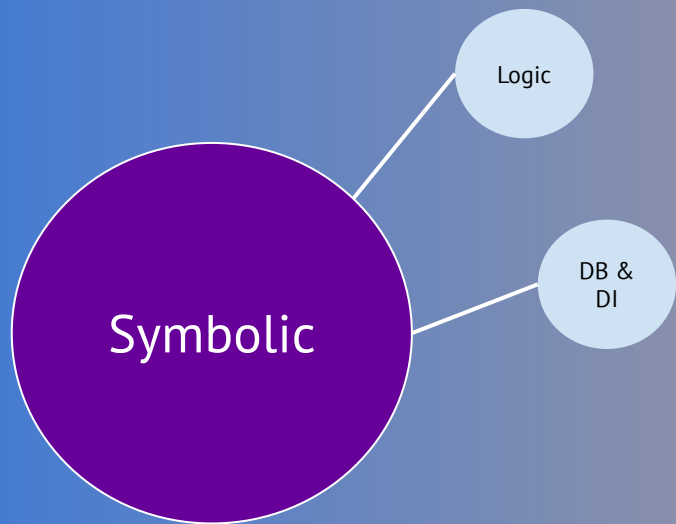


Placing Knowledge Graphs In Graph ML

On representation of Knowledge Graphs

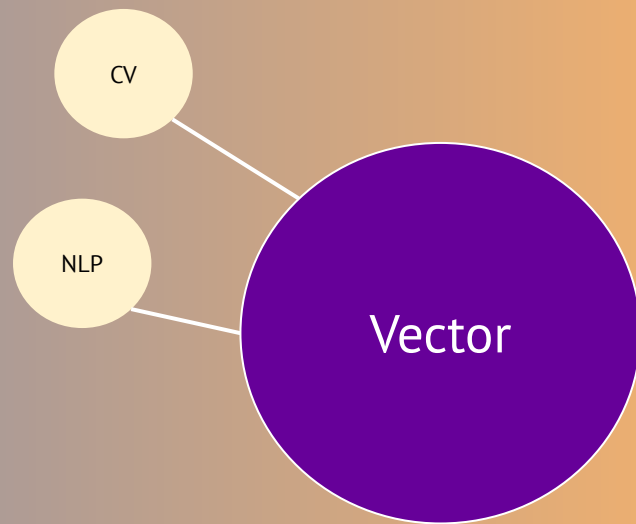


s, p, o
 $p(s, o)$
 (s, p, o)

Open-world assumption

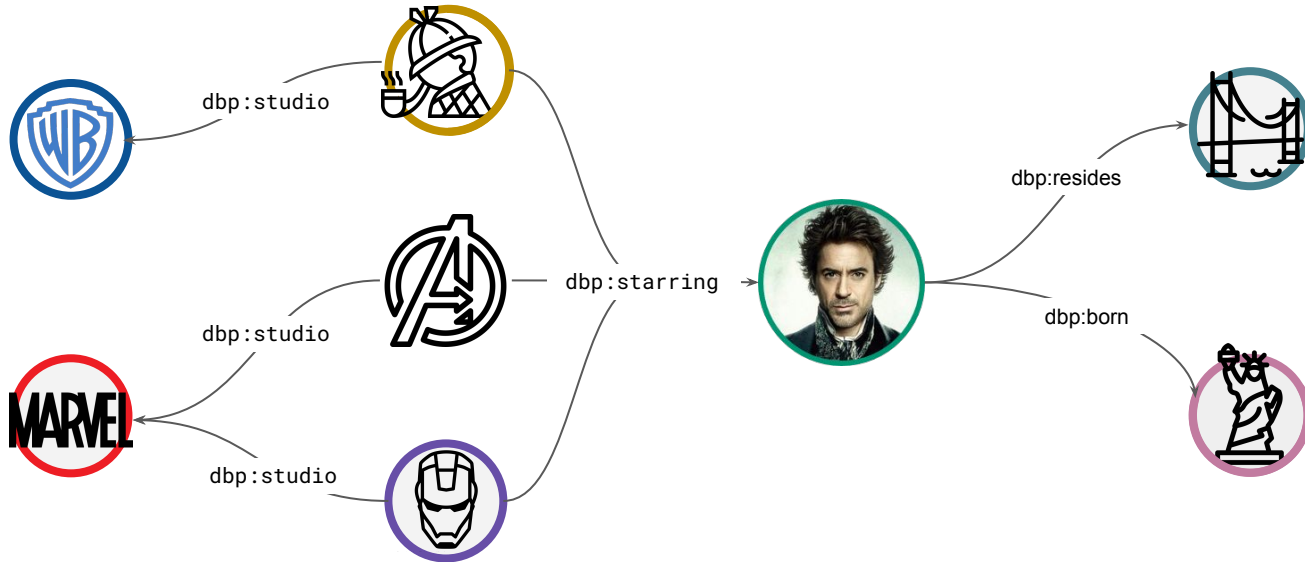
Closed-world assumption

Temporal / evolving



$s, p, o \in \mathbb{R}^d$

Vanilla Triples



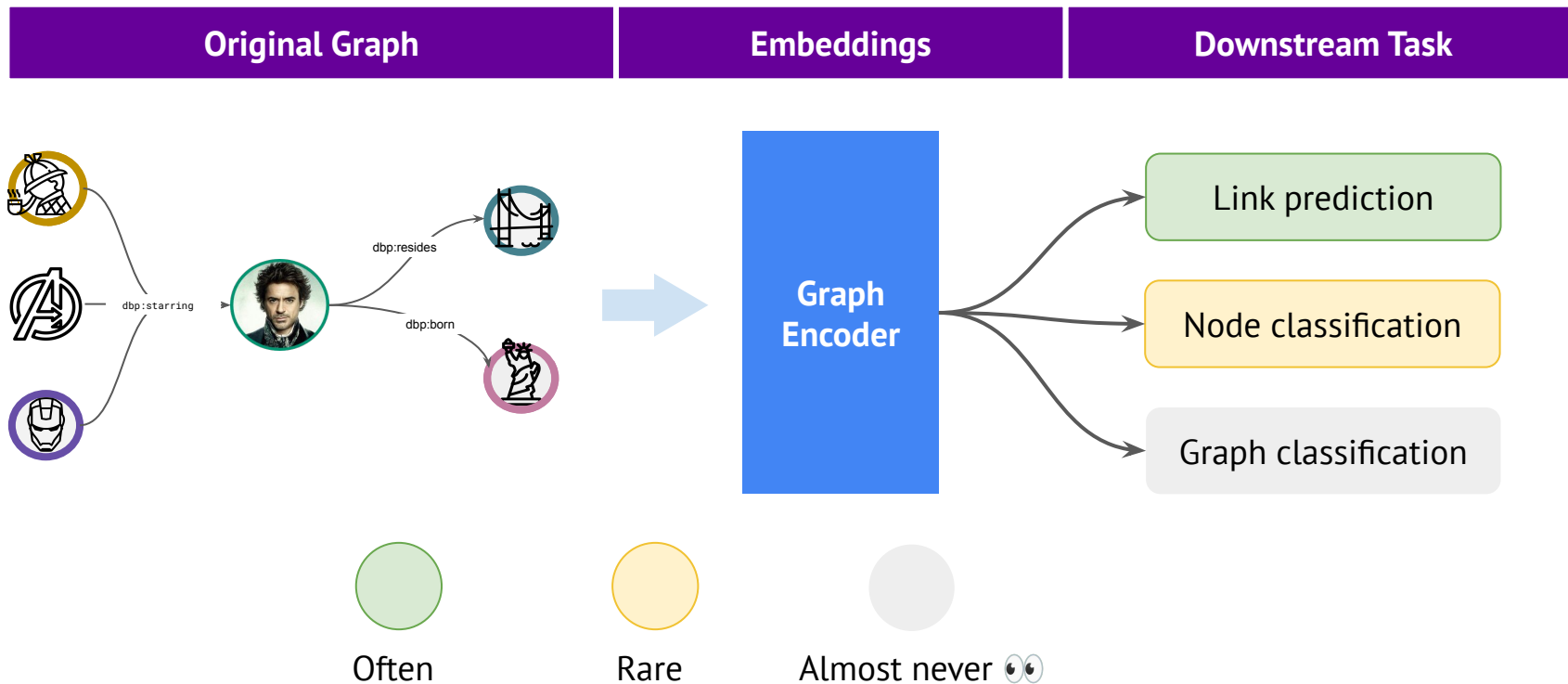
RDJ
 RDJ
 Sherlock_Holmes
 Sherlock_Holmes

dbp:resides SF .
 dbp:born NY .
 dbp:studio WB .
 dbp:starring RDJ .

Avengers
 Avengers
 Iron_Man
 Iron_Man

dbp:studio Marvel .
 dbp:starring RDJ .
 dbp:studio Marvel .
 dbp:starring RDJ .

KGs in Graph ML



KGs in Graph ML: Big Picture in \mathbb{R}^5

Transductive

Triples

Supervised

Unimodal

Small

Inductive

Hyper-relational

Unsupervised

Multimodal

Large (sampling)

SETTING

TASK

Link prediction

Node classification

Entity Matching

Query Embedding

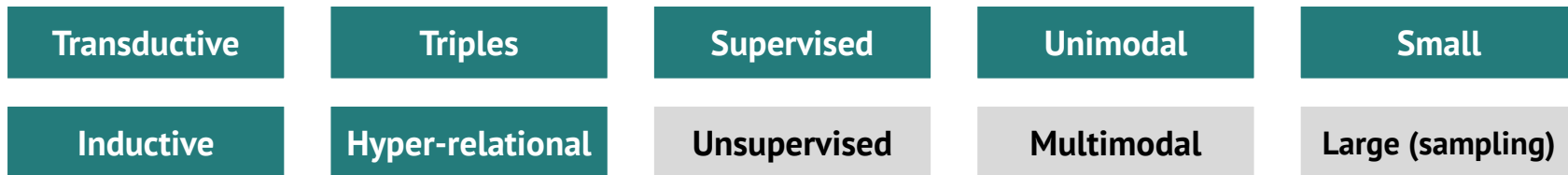
Theoretical Understanding

Graph Encoder

Generative Models

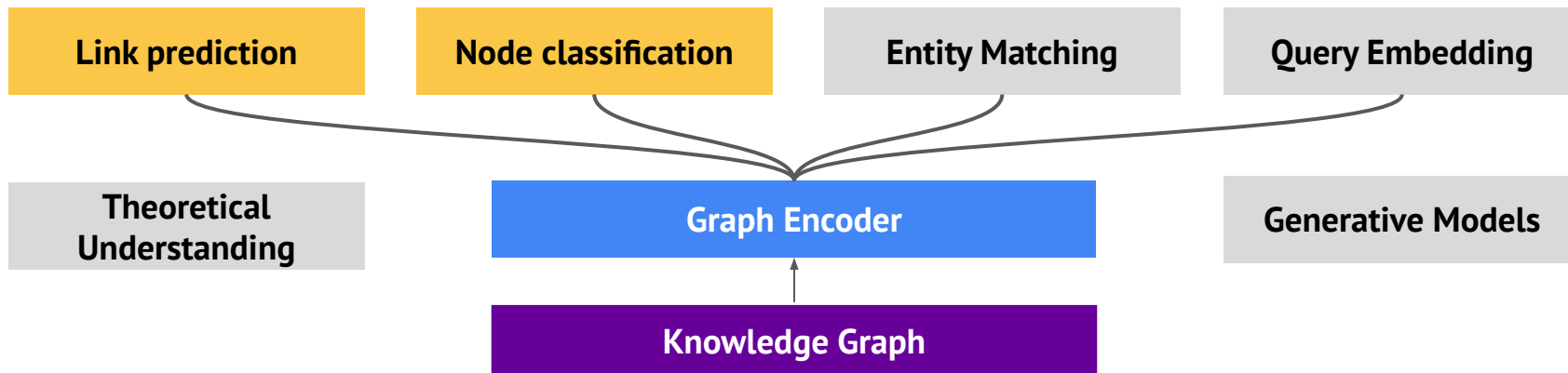
Knowledge Graph

Hyper-Relational KGs

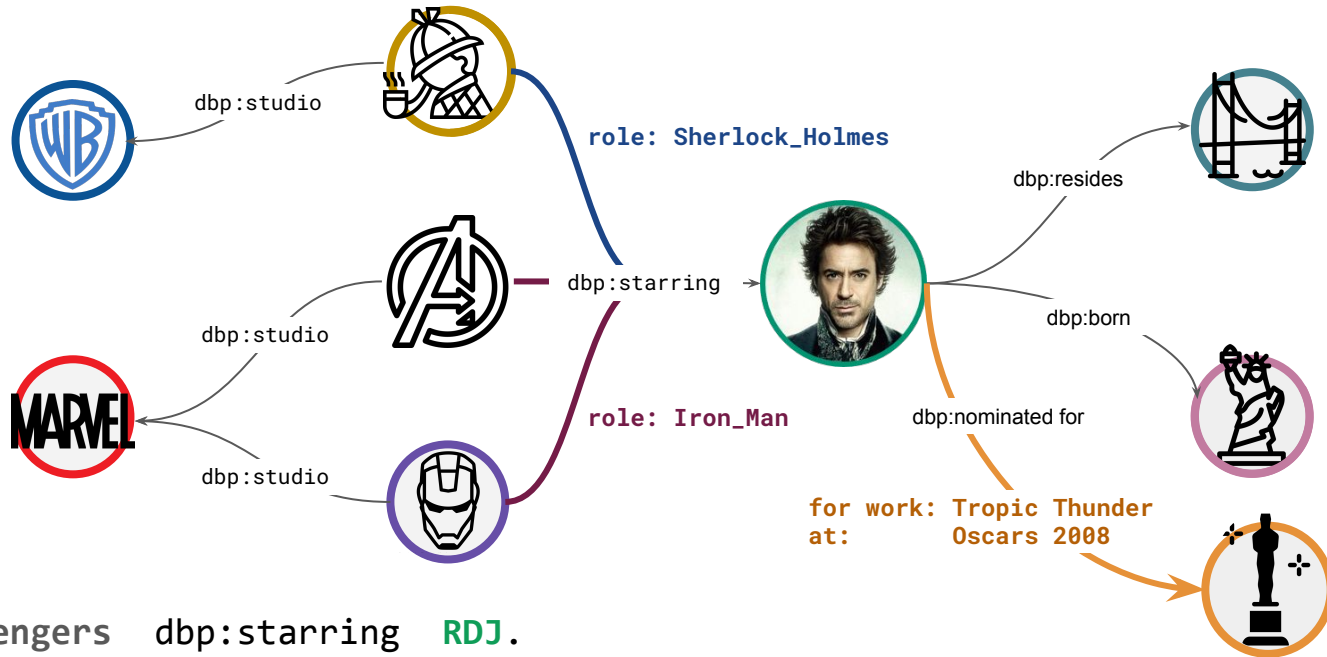


SETTING

TASK



Hyper-Relational RGS: RDF and SPARQL*

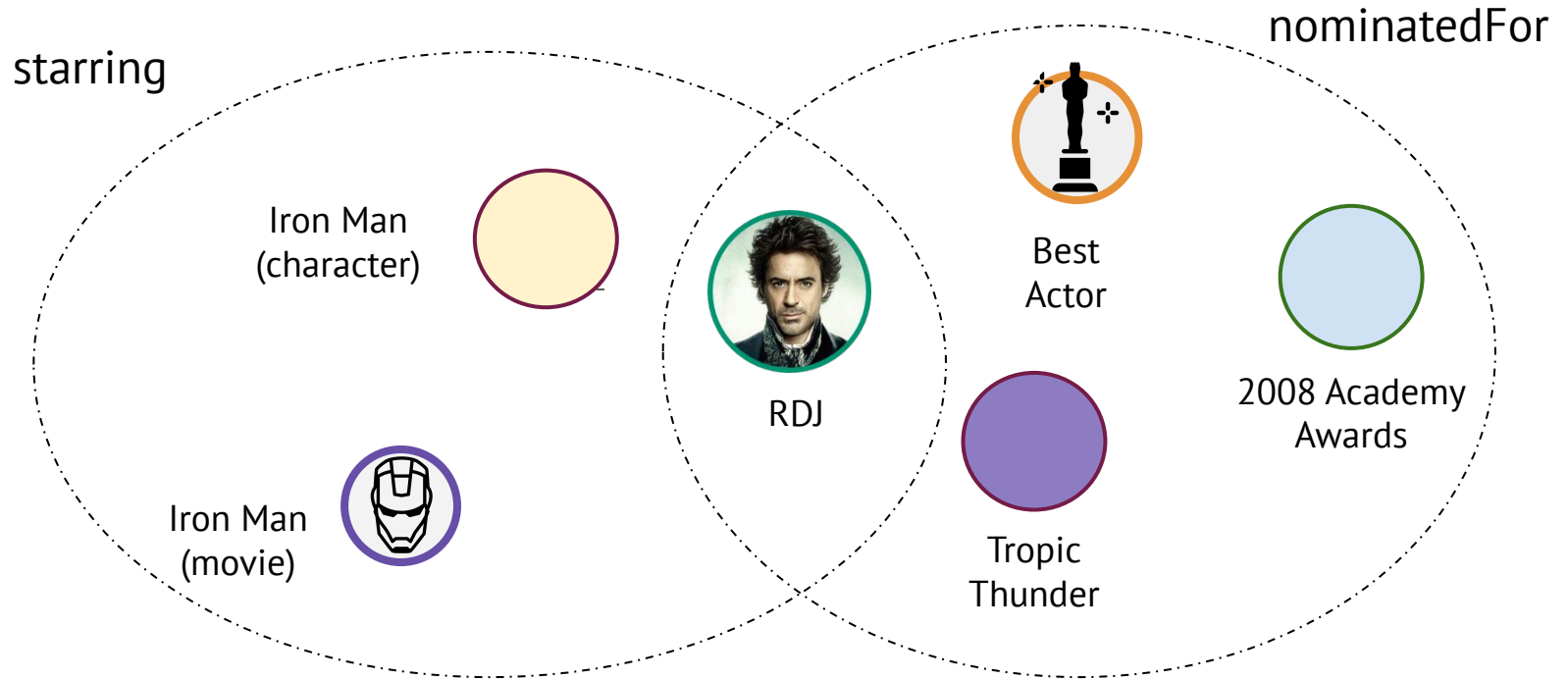


The_Avengers dbp:starring RDJ.

<< The_Avengers dbp:starring RDJ >> role Iron_Man .

<< RDJ dbp:nominated_for Oscar >> for_work Tropic_Thunder;
at Oscars_2008 .

Hyper-Relational \neq Hypergraphs



Immediate loss of the fine-grained predicates & e-r attribution

Multirelational GNN Encoders for KGs

$$\mathbf{h}_v^{(k)} = f \left(\sum_{u \in \mathcal{N}(v)} \mathbf{W}^{(k)} \mathbf{h}_u^{(k-1)} \right)$$

Vanilla GCN: no relations

$$\mathbf{h}_v^{(k)} = f \left(\sum_{(u,r) \in \mathcal{N}(v)} \mathbf{W}_r^{(k)} \mathbf{h}_u^{(k-1)} \right)$$

R-GCN [1]: a whole matrix W per relation

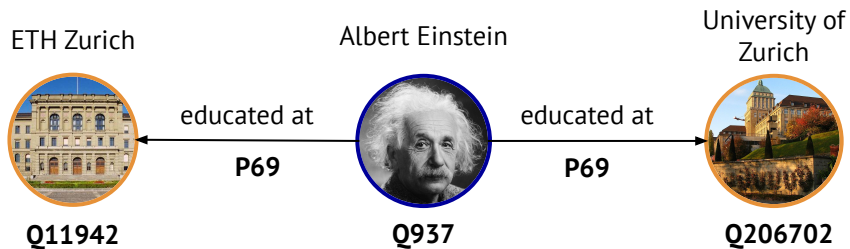
$$\mathbf{h}_v = f \left(\sum_{(u,r) \in \mathcal{N}(v)} \mathbf{W}_{\lambda(r)} \phi(\mathbf{x}_u, \mathbf{z}_r) \right)$$

CompGCN [2]: a vector \mathbf{z}_r per relation +
composition of (\mathbf{h}, \mathbf{r}) +
only 3 different W : input/output/self-loop

[1] Schlichtkrull et al. Modeling Relational Data with Graph Convolutional Networks. ESWC 2018

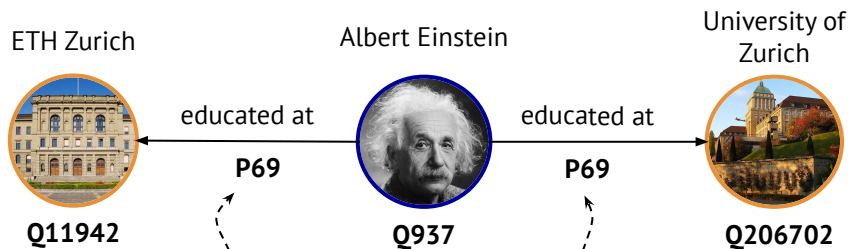
[2] Vashishth et al. Composition-Based Multi-Relational Graph Convolutional Networks. ICLR 2020

Embedding Hyper-Relational KGs



A. Triple-based facts

B. Hyper-relational facts



Academic degree (P512):
Bachelor (Q787674)
Academic major (P812):
Mathematics (Q853077)

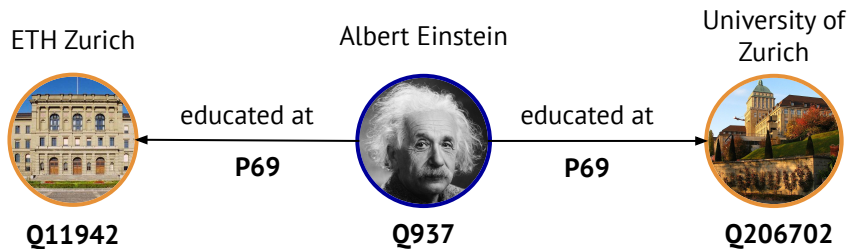
Academic degree (P512):
Doctorate (Q849697)
Academic major (P812):
Physics (Q413)

$$h_v = f \left(\sum_{(u,r) \in \mathcal{N}(v)} W_{\lambda(r)} \phi(x_u, z_r) \right)$$

?

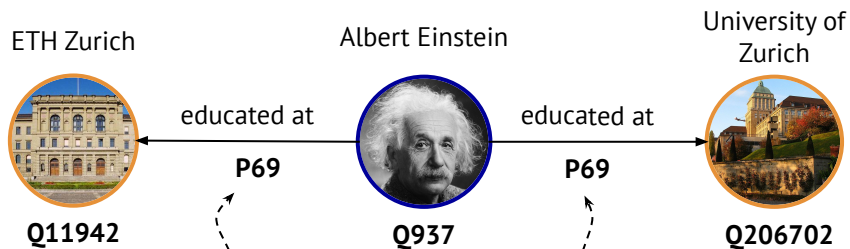
- Qualifying relations and entities can be used as main terms in other facts
- Not all facts might have qualifiers

Embedding Hyper-Relational KGs



A. Triple-based facts

B. Hyper-relational facts



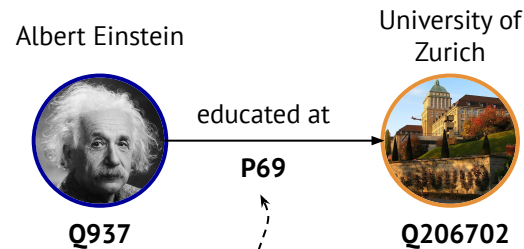
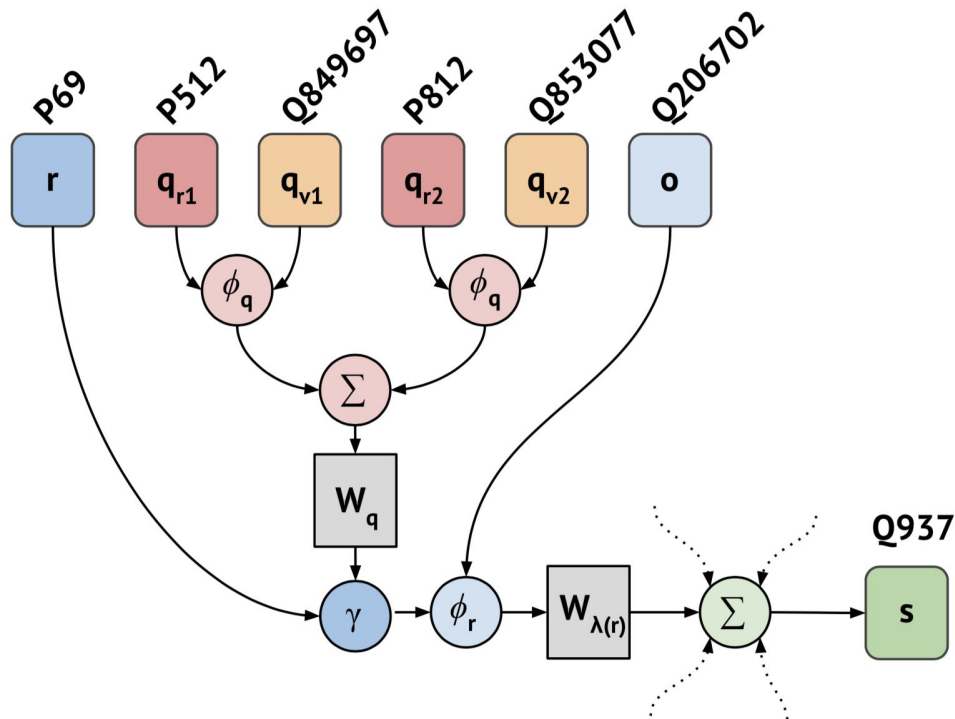
Academic degree (P512):
Bachelor (Q787674)
Academic major (P812):
Mathematics (Q853077)

Academic degree (P512):
Doctorate (Q849697)
Academic major (P812):
Physics (Q413)

$$\mathbf{h}_v = f \left(\sum_{(u,r) \in \mathcal{N}(v)} \mathbf{W}_{\lambda(r)} \phi(\mathbf{x}_u, \mathbf{z}_r) \right)$$

$$\mathbf{h}_v = f \left(\sum_{(u,r) \in \mathcal{N}(v)} \mathbf{W}_{\lambda(r)} \phi_r(\mathbf{h}_u, \gamma(\mathbf{h}_r, \mathbf{h}_q)_{vu}) \right)$$

Embedding Hyper-Relational KGs



$$\mathbf{h}_v = f \left(\sum_{(u,r) \in \mathcal{N}(v)} \mathbf{W}_{\lambda(r)} \phi_r(\mathbf{h}_u, \gamma(\mathbf{h}_r, \mathbf{h}_q)_{vu}) \right)$$

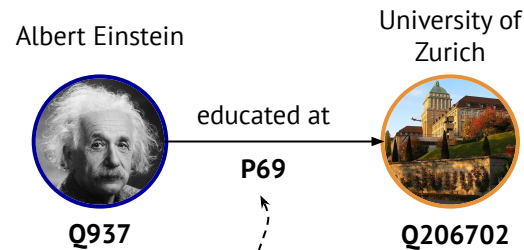
Encoding Hyper-Relational KGs

Sparse Triple Representation

<i>s</i>	Q937
<i>o</i>	Q206702
<i>r</i>	P69
<i>index</i>	<i>k</i>	<i>k+1</i>	<i>k+2</i>

Sparse Qualifier Representation

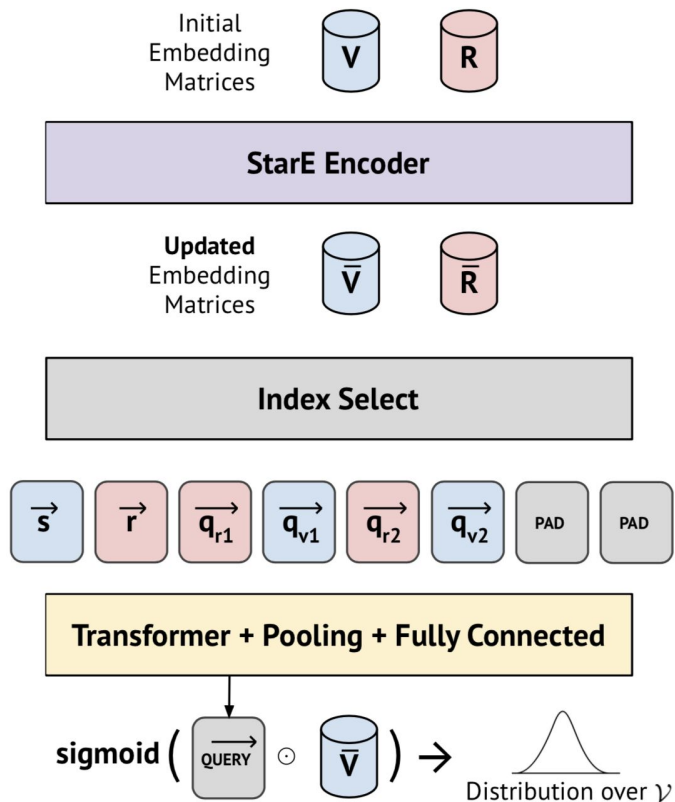
<i>index</i>	<i>k</i>	<i>k</i>	...
<i>qr</i>	P812	P512	...
<i>qv</i>	Q413	Q849697	...



Academic degree (P512):
 Doctorate (Q849697)
 Academic major (P812):
 Physics (Q413)

$$O(|\mathcal{E}| + |\mathcal{Q}|) \quad \text{Space complexity}$$

Decoders for Downstream Tasks

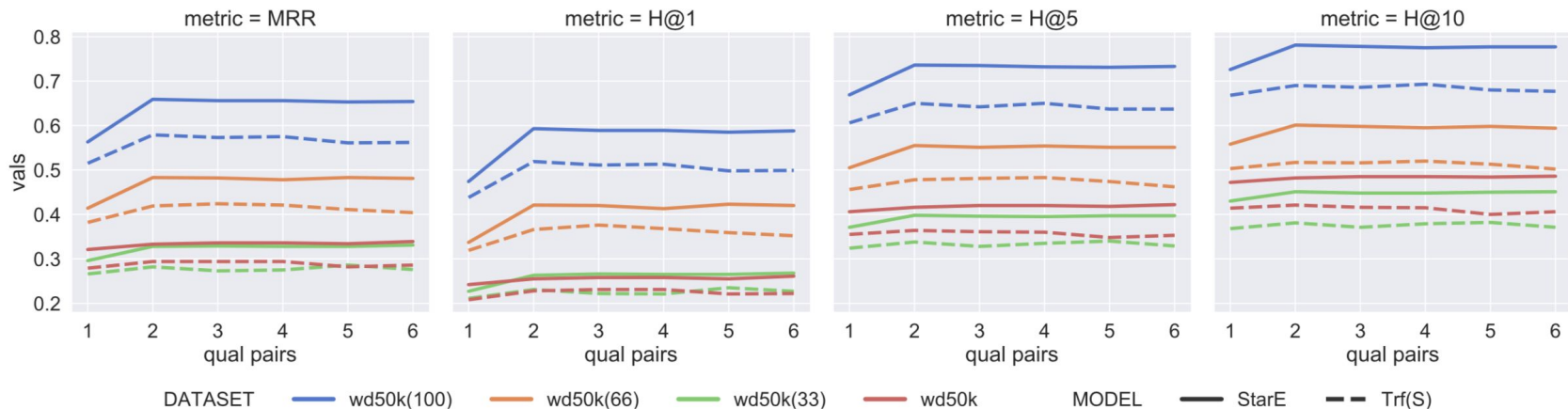


← A decoder for link prediction

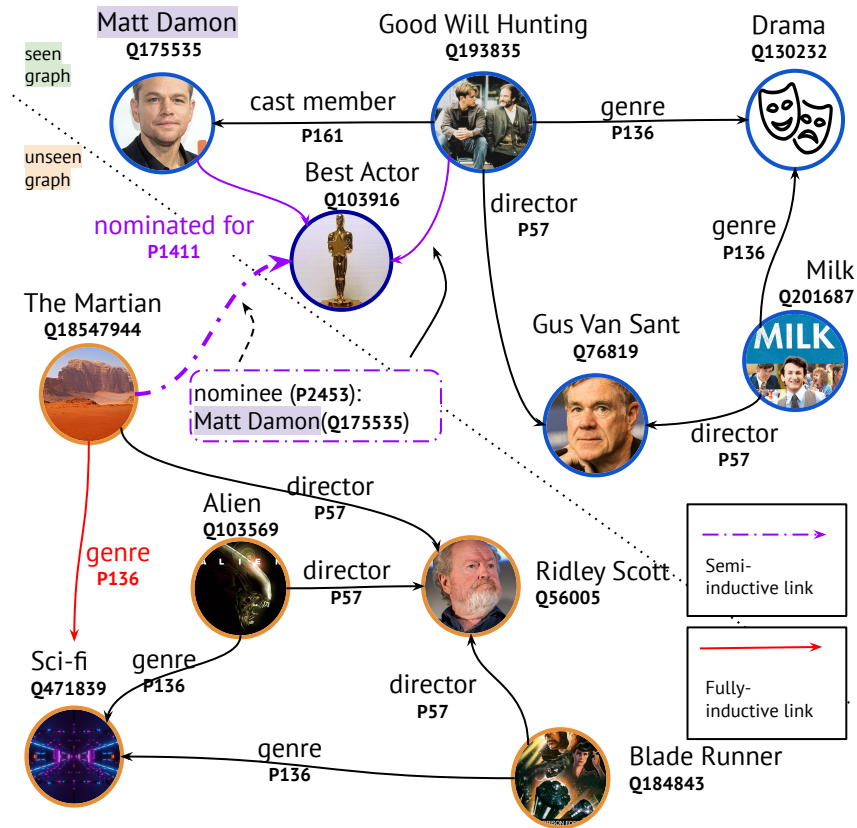
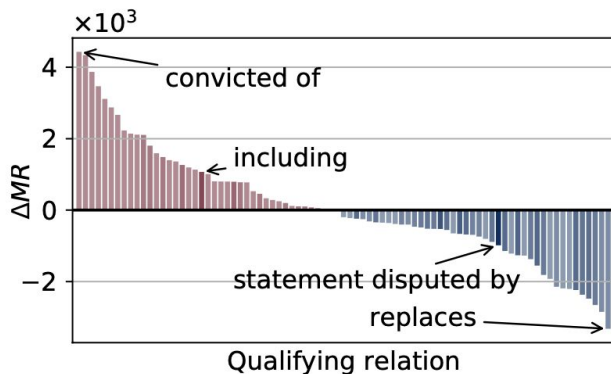
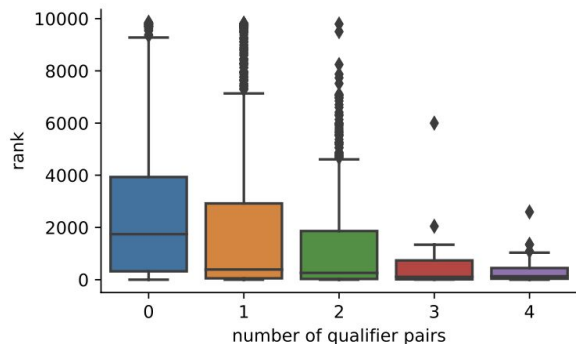
- Datasets issue:
 - No such graphs in OGB
 - Very few LP datasets

Hyper-Relational KGs: Link Prediction

Exp #	Dataset → Method ↓	WD50K			WD50K (33)			WD50K (66)			WD50K (100)		
		MRR	H@1	H@10	MRR	H@1	H@10	MRR	H@1	H@10	MRR	H@1	H@10
4	Baseline (Transformer (T))	0.275	0.207	0.404	0.218	0.158	0.334	0.270	0.197	0.417	0.351	0.261	0.530
4	STARE (T) + Transformer(T)	0.308	0.228	0.465	0.246	0.173	0.388	0.297	0.212	0.470	0.380	0.276	0.584
1,2,4	Baseline (Transformer (H))	0.286	0.222	0.406	0.276	0.227	0.371	0.404	0.352	0.502	0.562	0.499	0.677
1,2,4	STARE (H) + Transformer(H)	0.349	0.271	0.496	0.331	0.268	0.451	0.481	0.420	0.594	0.654	0.588	0.777



Inductive Scenarios



Big Picture in \mathbb{R}^5

Transductive

Triples

Supervised

Unimodal

Small

Inductive

Hyper-relational

Unsupervised

Multimodal

Large (sampling)

SETTING

TASK

Link prediction

Node classification

Entity Matching

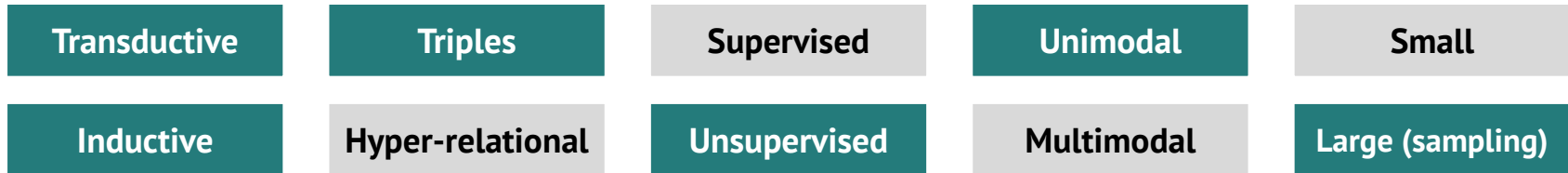
Query Embedding

Theoretical
Understanding

Graph Encoder

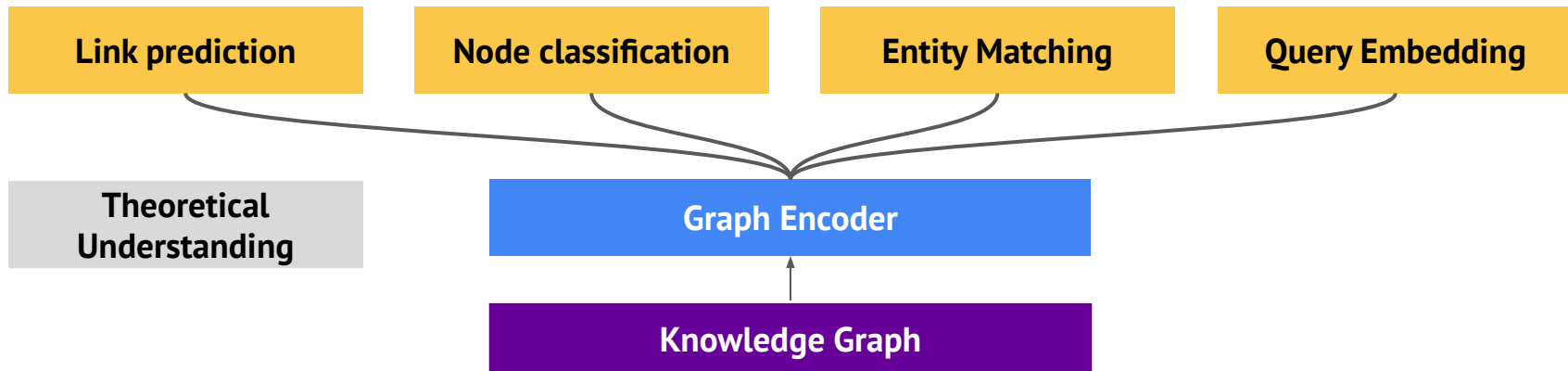
Knowledge Graph

Big Picture in \mathbb{R}^5 - Scaling Up

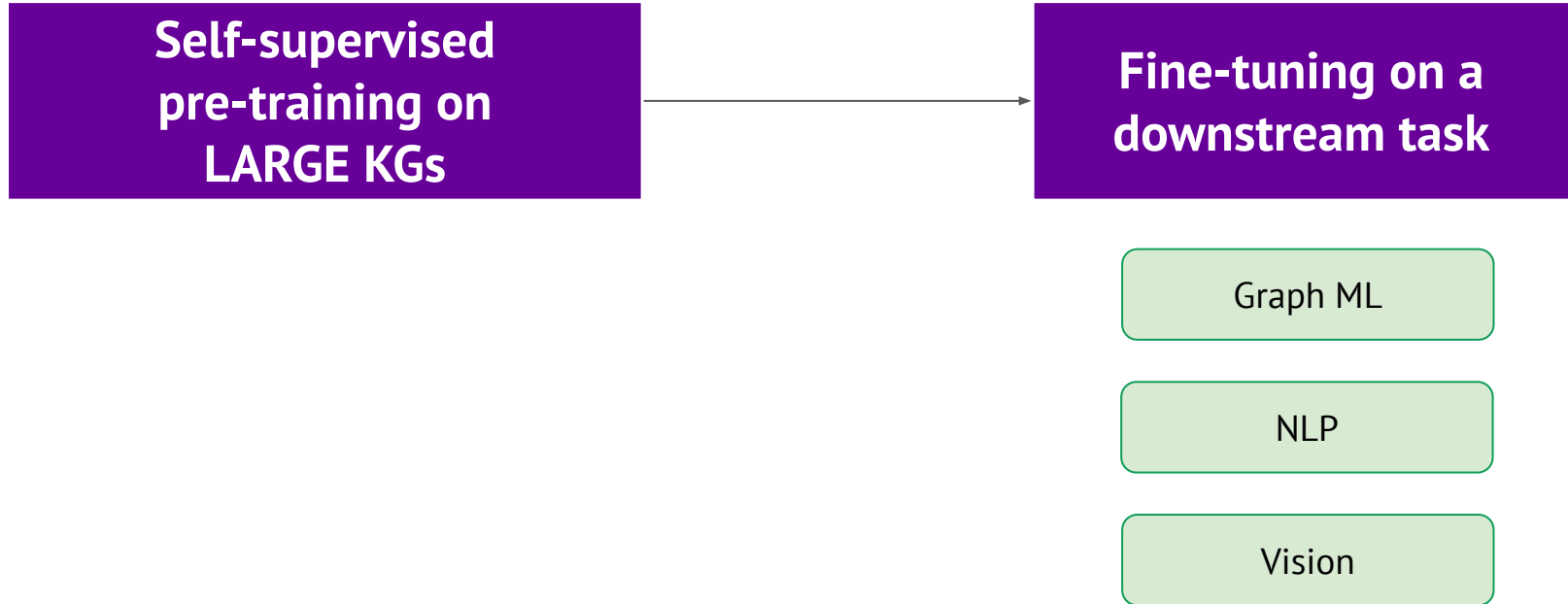


SETTING

TASK



The ImageNet Moment for KGs



The ImageNet Moment for KGs

Self-supervised
pre-training on
LARGE KGs



Fine-tuning on a
downstream task

Wikidata: 100M nodes
Embs: [100M, dim] ?

 PyTorch BigGraph

~200 GB



Graph ML

NLP

Vision

OGB WikiKG: Just 2.5M nodes

Leaderboard for [ogbl-wikikg2](#)

The MRR score on the test and validation sets. The higher, the better.

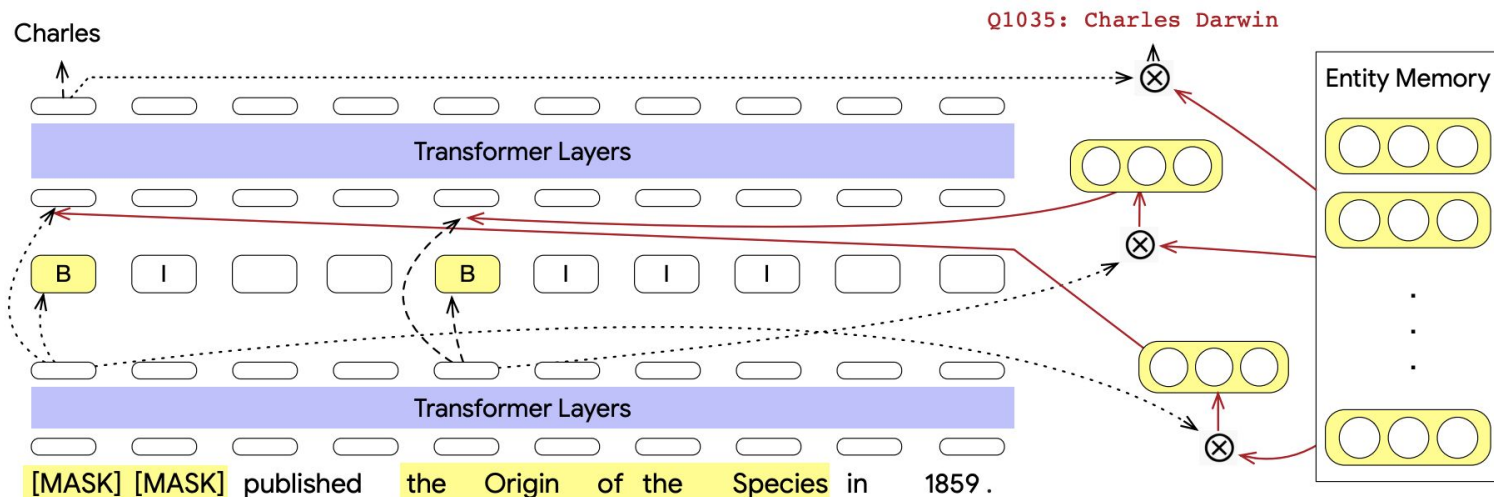
Package: $\geq 1.2.4$

Deprecated [ogbl-wikikg](#) leaderboard can be found [here](#).

BERT-Large is ~340M params

Rank	Method	Test MRR	Validation MRR	Contact	References	#Params	Hardware	Date
1	PairRE (200dim)	0.5208 \pm 0.0027	0.5423 \pm 0.0020	Linlin Chao	Paper , Code	500,334,800	Tesla P100 (16GB GPU)	Jan 28, 2021
2	RotatE (250dim)	0.4332 \pm 0.0025	0.4353 \pm 0.0028	Hongyu Ren – OGB team	Paper , Code	1,250,435,750	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021
3	TransE (500dim)	0.4256 \pm 0.0030	0.4272 \pm 0.0030	Hongyu Ren – OGB team	Paper , Code	1,250,569,500	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021
4	ComplEx (250dim)	0.4027 \pm 0.0027	0.3759 \pm 0.0016	Hongyu Ren – OGB team	Paper , Code	1,250,569,500	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021

Explicit embeddings in LMs



Wikidata-scale (100M nodes, 2B edges):

[100M, 200d]

Sparsifying / Tokenizing KGs

BERT-Large (340M)

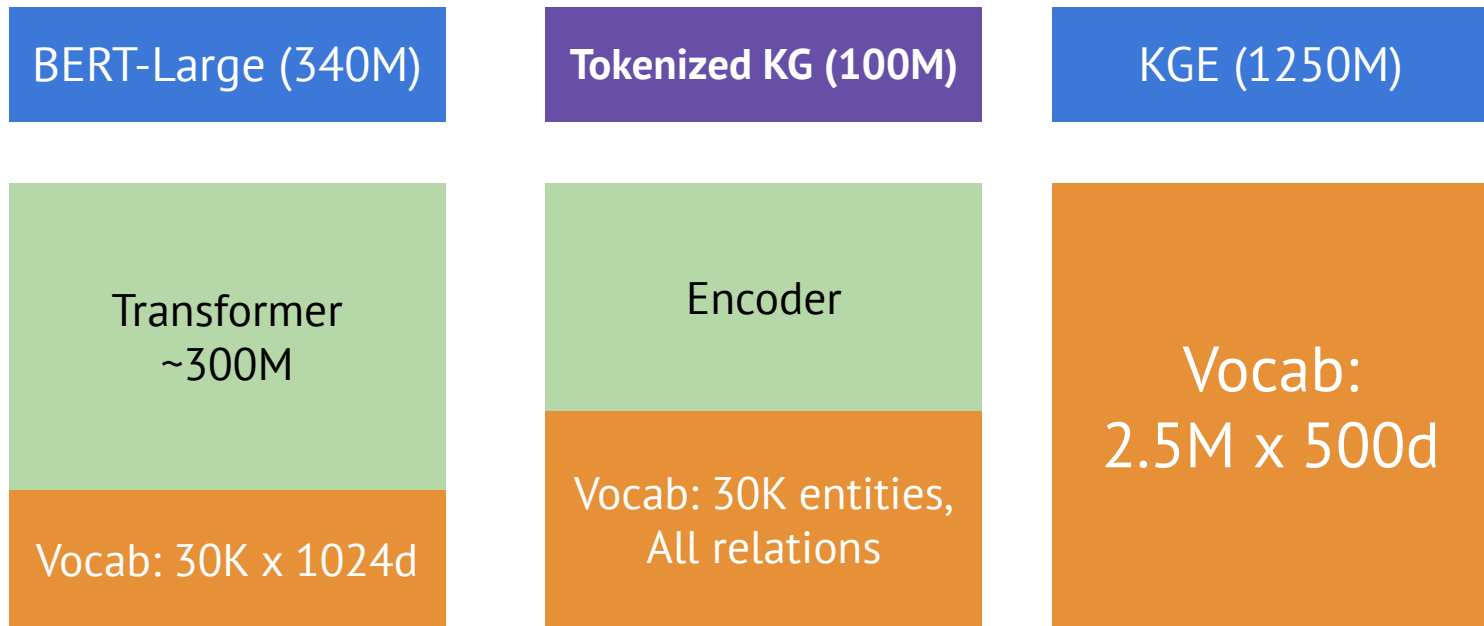
Transformer
~300M

Vocab: 30K x 1024d

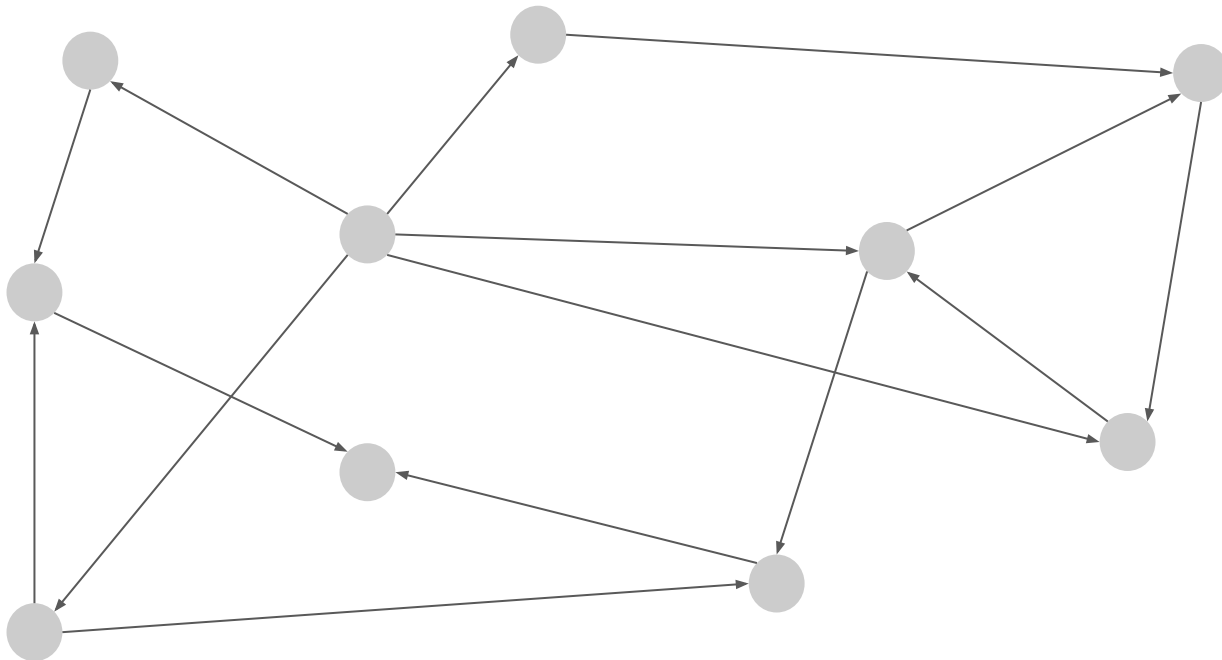
KGE (1250M)

Vocab:
2.5M x 500d

Sparsifying / Tokenizing KGs

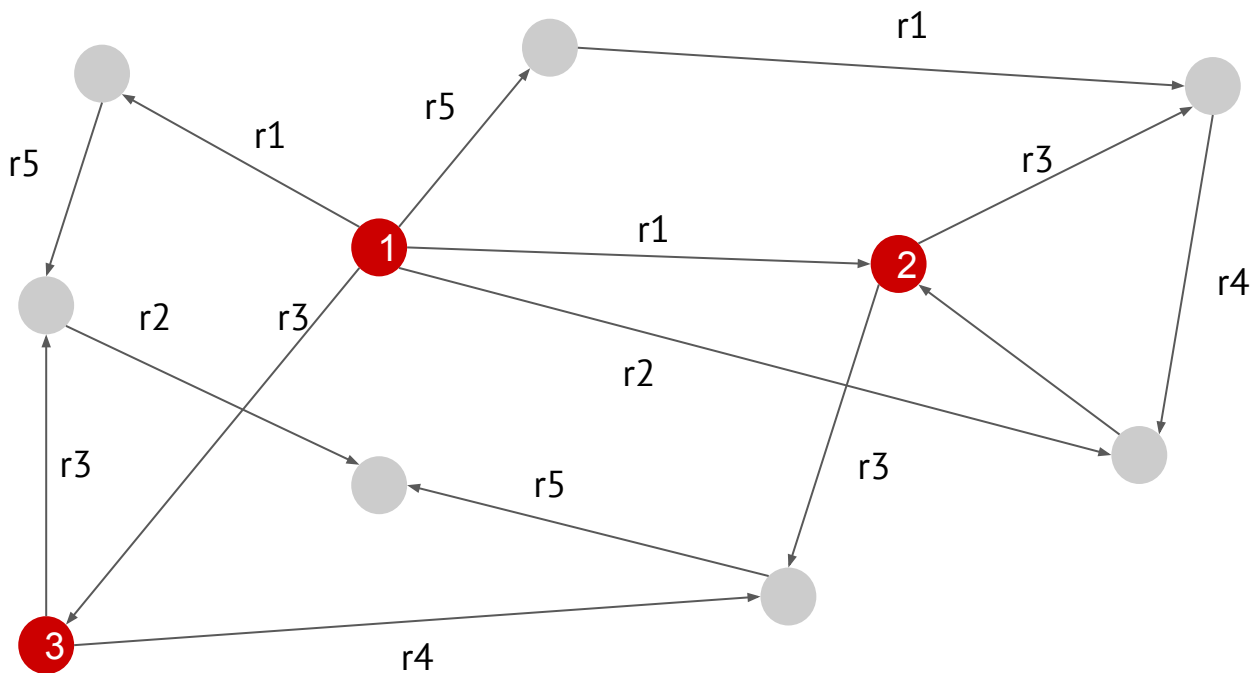


Sparsifying / Tokenizing KGs

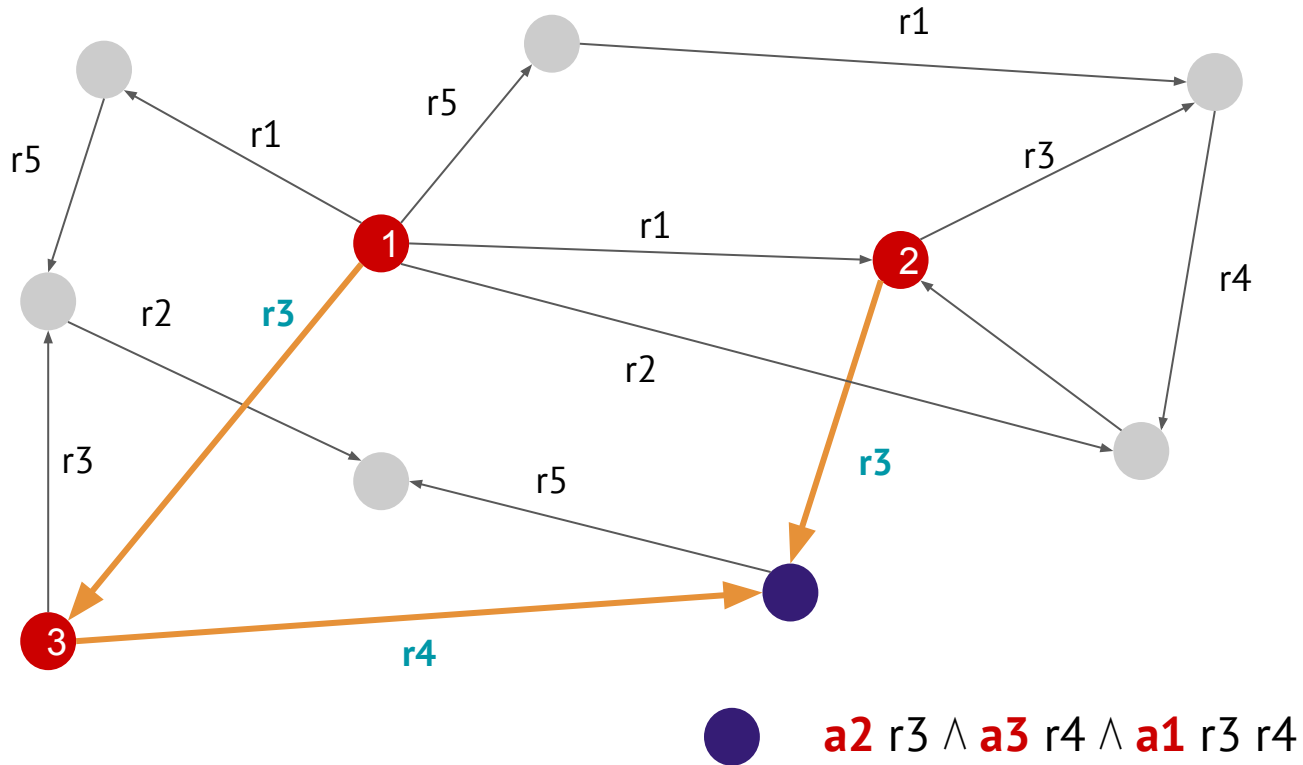


Sparsifying / Tokenizing KGs

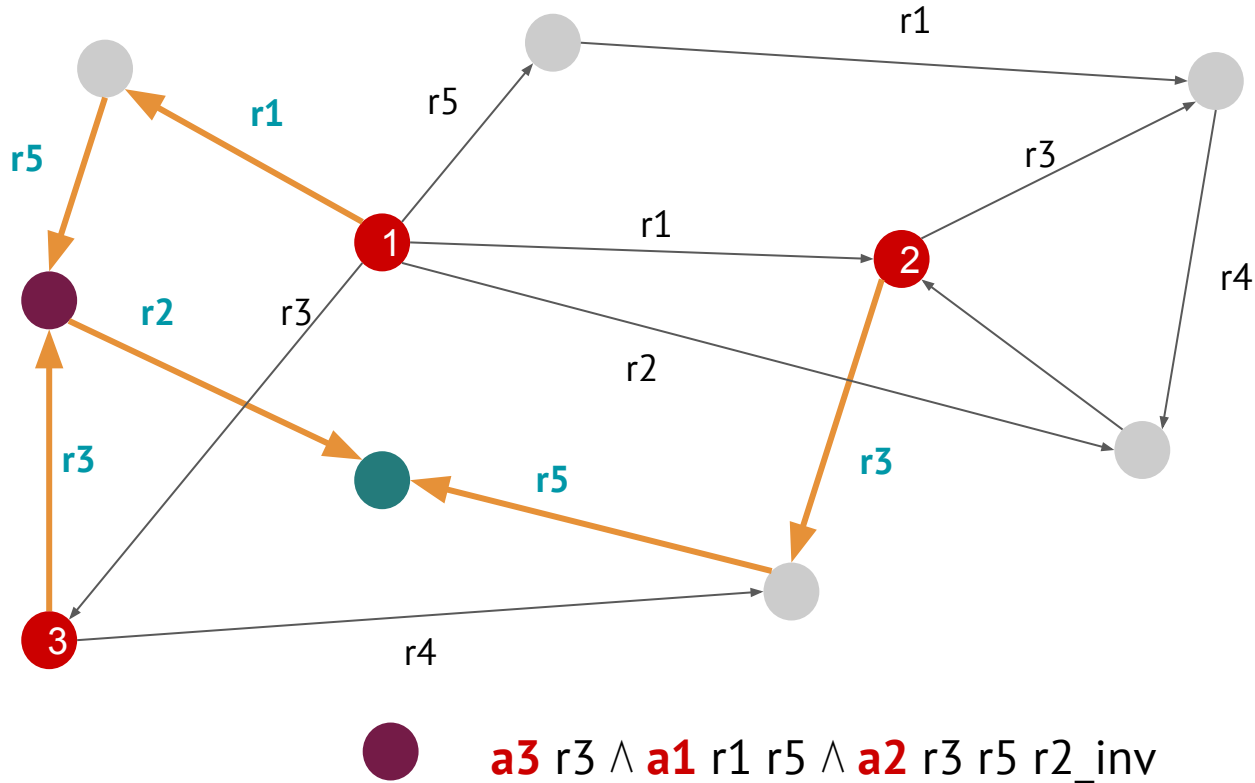
- **k anchors**
- **Conjunctive query of anchor paths**



Sparsifying / Tokenizing KGs



Sparsifying / Tokenizing KGs



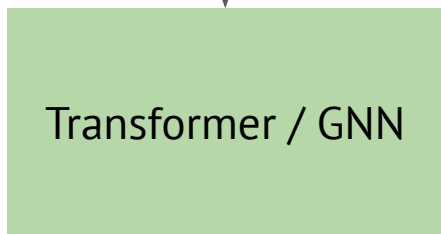
Sparsifying / Tokenizing KGs

● $a_3 r_3 \wedge a_1 r_1 r_5 \wedge a_2 r_3 r_5 r_2i$

[CLS] | $a_3 r_3$ | [MASK] $r_1 r_5$ | $a_2 r_3 r_5 r_2i$

[CLS] | $a_3 r_3$ | $a_1 r_1 r_5$ | $a_2 r_3 r_5 r_2i$

[CLS] | $a_1 r_1 r_5$ | $a_3 r_3$ | $a_2 r_3 r_5 r_2i$

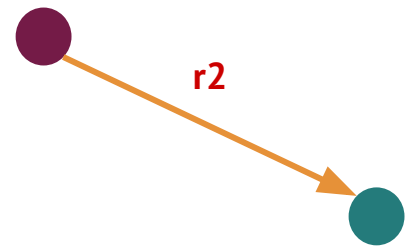


MLM loss



BYOL loss

Fine-tuning decoders



● [CLS] | a3 r3 | a1 r1 r5 | a2 r3 r5 r2i

● [CLS] | a3 r4 r1 | a2 r1

Old TransE

$$[h] + r2 - [t]$$

● hr [CLS] | a3 r3 r2 | a1 r1 r5 r2 | a2 r3 r5 r2i r2

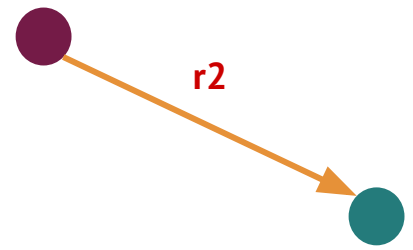
● [CLS] | a3 r4 r1 | a2 r1

$$hr \cdot t$$

$$\frac{hr \cdot t}{||hr|| \cdot ||t||}$$

New Cosine

Positional encodings



hr	[CLS]		a3	r3	r2		a1	r1	r5	r2		a2	r3	r5	r2i	r2	
	0		1	2	3		1	2	3	4		1	2	3	4	5	Op1
	0		1	1	1		2	2	2	2		3	3	3	3	3	Op2
	0		2	p	p		3	p	p	p		4	p	p	p	p	Op3
	0		1	2	3		4	5	6	7		8	9	10	11	12	Op4

Even more!

- The ImageNet moment for KGs
- Compositional generalization with approximate query embedding
- KGs + links to the spectral theory
- Logical expressiveness of triple-based and hyper-relational KGs