

Stevie Ray
Vaughan



Q202937

Fender
Stratocaster



Q208173

Norwegian
Black Metal



Q2296521

Inductive Graph Reasoning without Node Features

Michael Galkin
Postdoctoral Fellow @ Mila & McGill



John Mayer

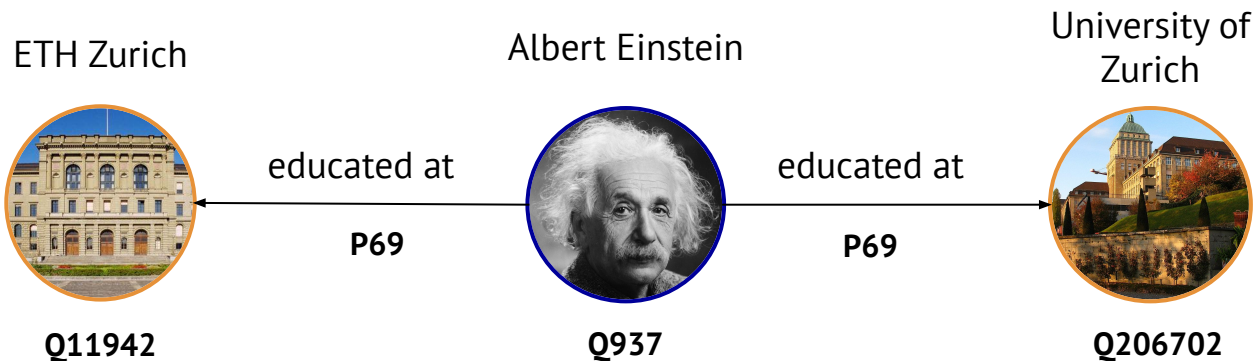


Q215215

Plan

- **Graph Reasoning Tasks**
- Featurization via Tokenization: NodePiece
- Featurization via Labeling Trick:
Neural Bellman-Ford and GNN-QE
- Past, Today, Future

Triple-based Knowledge Graphs



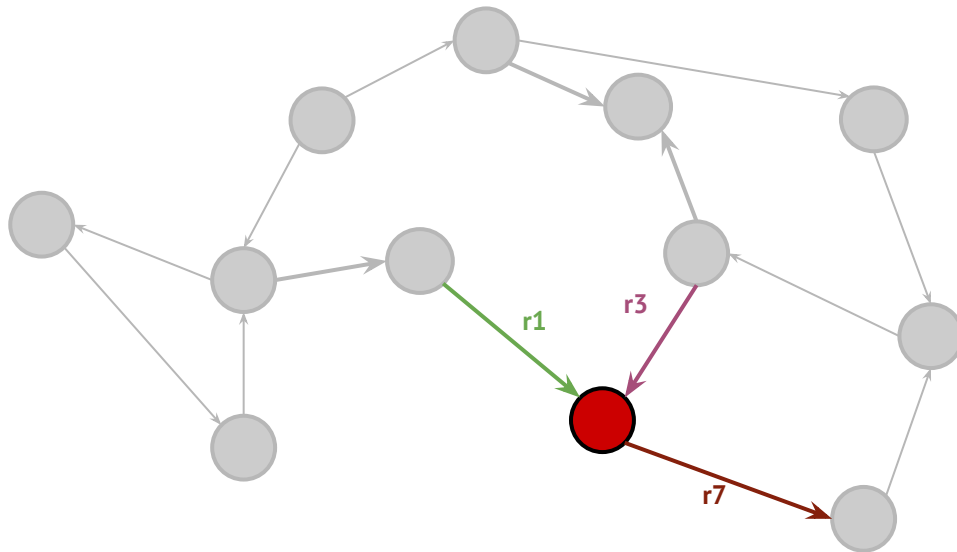
RDF

Albert Einstein
Albert Einstein

educatedAt
educatedAt

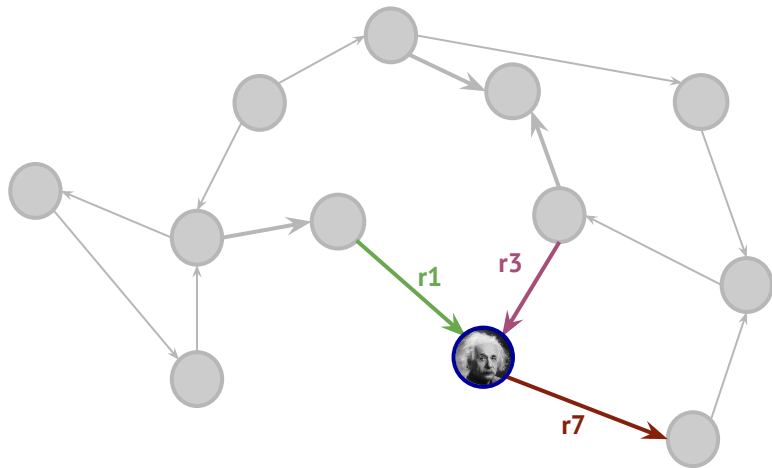
University of Zurich .
ETH Zurich .

Knowledge Graphs: Setup



- Directed graphs
- Explicit relation types (learnable edge features)
- Input node features are **not** given

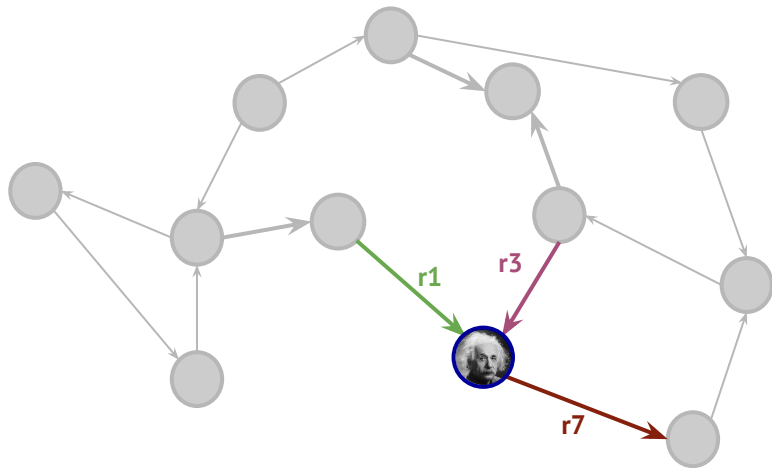
Graph Reasoning Tasks



- Node Classification

$$p(\text{type}(s) \mid \text{Einstein})$$

Graph Reasoning Tasks



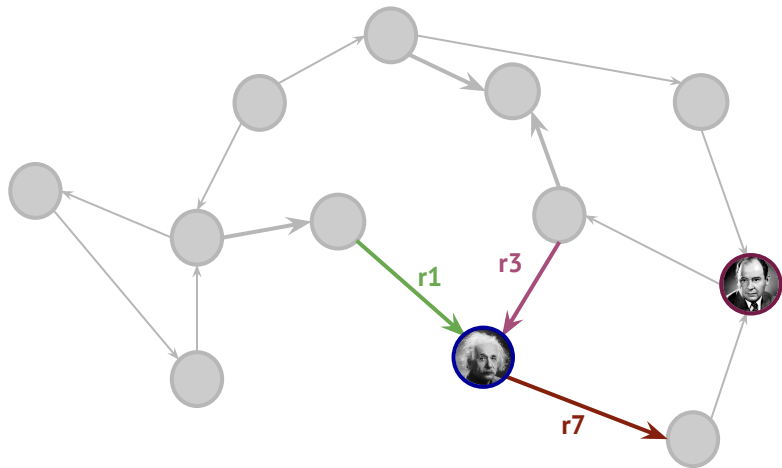
- Node Classification

$$p(\text{type}(s) \mid \text{img}(s))$$

- Simple Link Prediction

$$\text{img}(h) \xrightarrow{\text{educated at}} ? \quad p(\text{tail} \mid \text{head}, \text{relation})$$

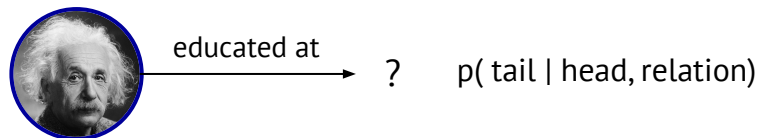
Graph Reasoning Tasks



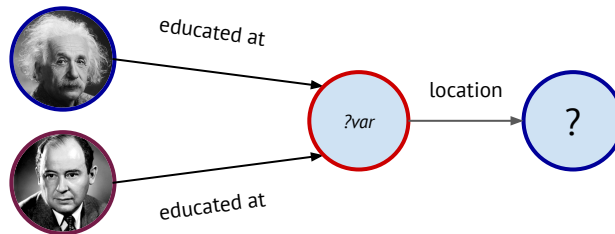
- Node Classification

$$p(\text{type}(s) \mid \text{img}(s))$$

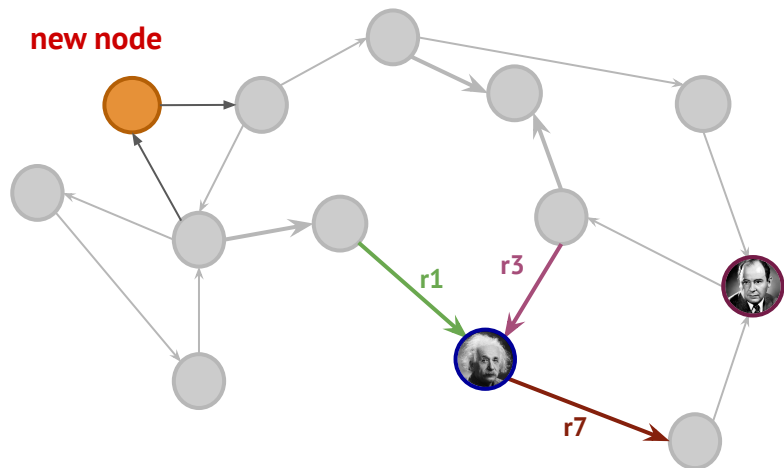
- Simple Link Prediction



- Complex Query Answering



Inductive Graph Reasoning Tasks

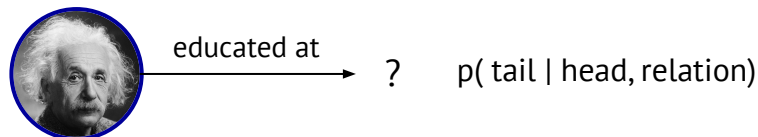


Extend the same tasks to **new, unseen** nodes arriving at inference time

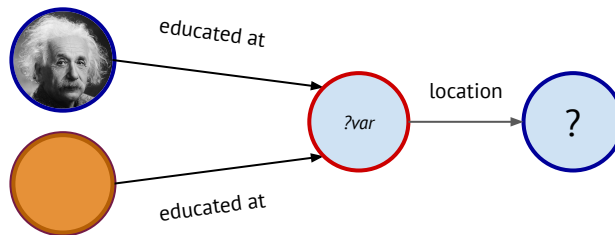
- Node Classification

$$p(\text{type}(s) \mid \text{orange node})$$

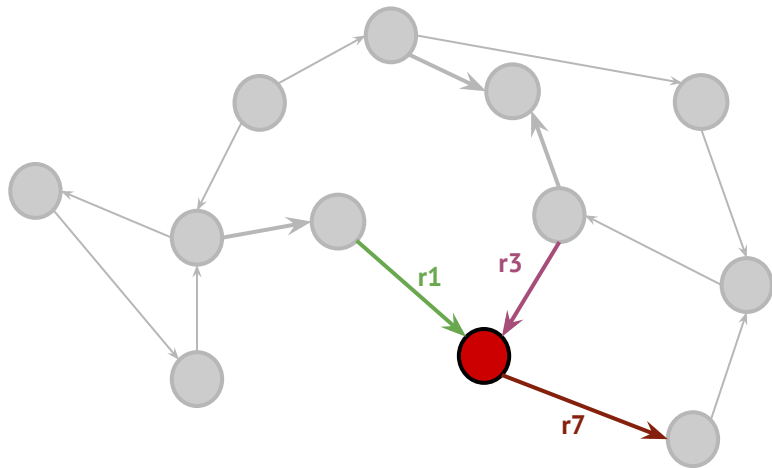
- Simple Link Prediction



- Complex Query Answering



Knowledge Graphs: Setup

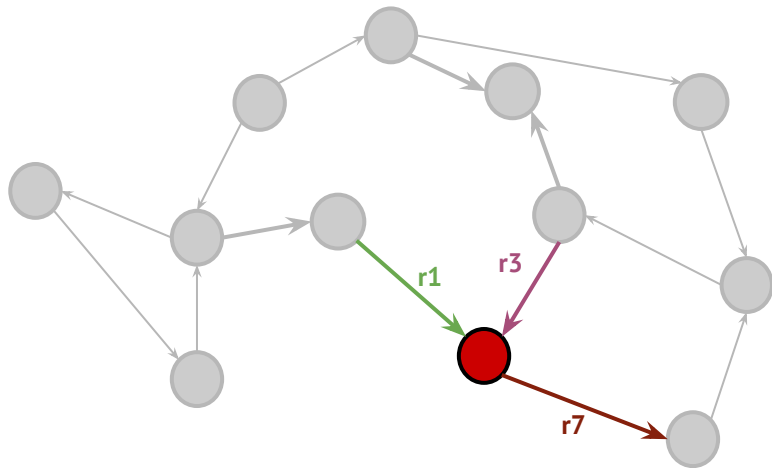


Any GNN-based pipeline needs features:

$$X' = \text{GNN}(X, A, W)$$

- Input node features are not given

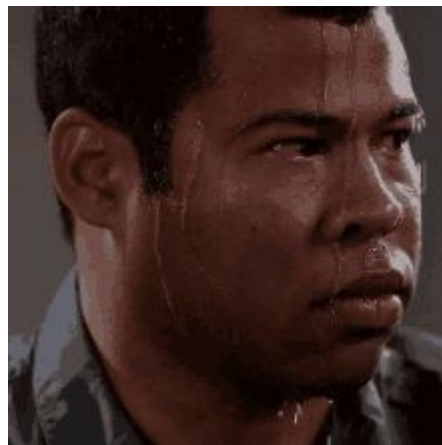
Knowledge Graphs: Setup



Any GNN-based pipeline needs features:

$$X' = \text{GNN}(X, A, W)$$

- Input node features are not given
- How do we get inductive features?



Brief History of Transductive Learning: 2011 -

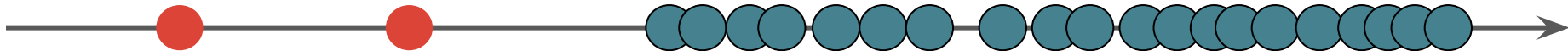
RESICAL

[Nickel et al, ICML 2011]

TransE

[Bordes et al, NeurIPS 2013]

100+ KG embedding models since then 🤖



Brief History of Transductive Learning: 2011 -

Transductive

Triples

Supervised

RESCAL

[Nickel et al, ICML 2011]

TransE

[Bordes et al, NeurIPS 2013]

100+ KG embedding models since then 🤖

Link Prediction on FB15k-237

No
substantial
progress since
2018

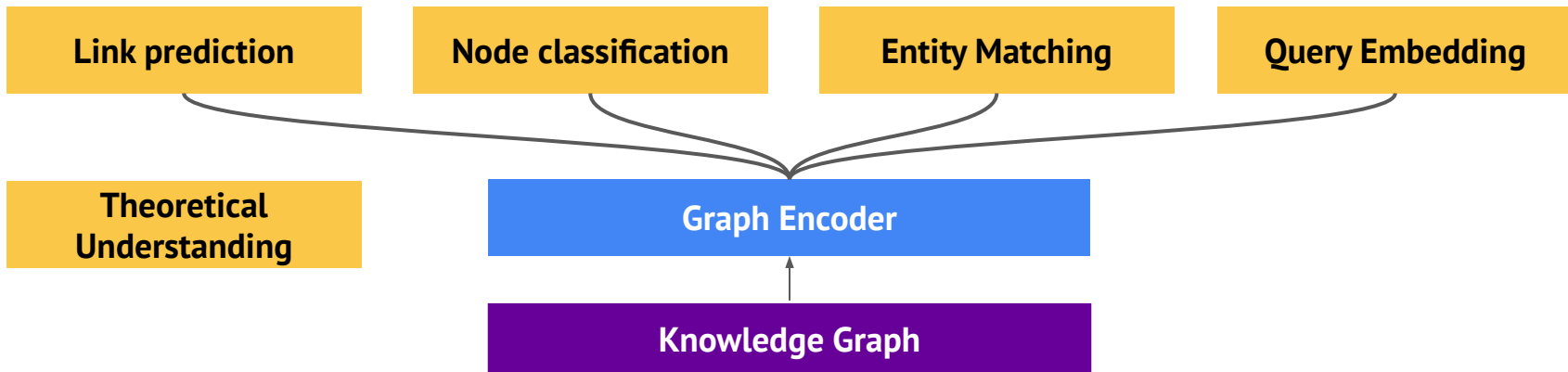


Big Picture in \mathbb{R}^5

Transductive	Triples	Supervised	Unimodal	Small
Inductive	Hyper-relational	Unsupervised	Multimodal	Large (sampling)

SETTING

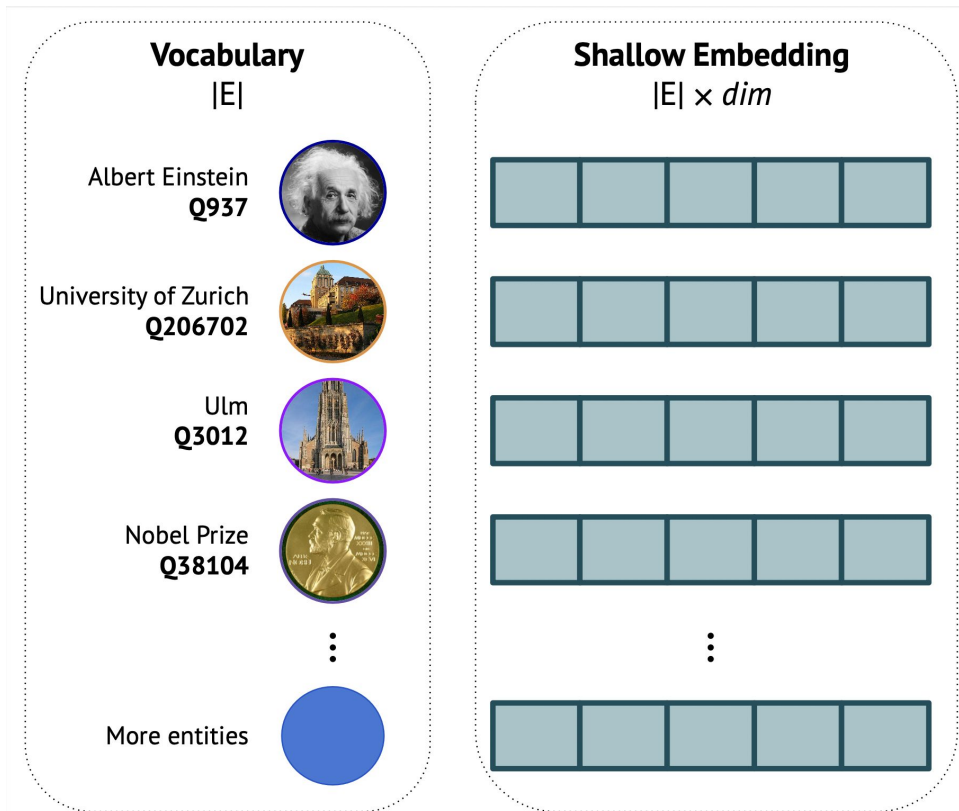
TASK



Shallow Embedding

Looks like a
Representation
Learning challenge 🤔

Can we do better?



Transductive vs Inductive

Shallow
embeddings

Transductive

Inductive

Training

Vocab



Inference



New, unseen nodes (entities)

- Added to the seen graph
- Completely new inference graph

OGB WikiKG: Just 2.5M nodes (June'21)

Leaderboard for [ogbl-wikikg2](#)

The MRR score on the test and validation sets. The higher, the better.

Package: $\geq 1.2.4$

Deprecated [ogbl-wikikg](#) leaderboard can be found [here](#).

BERT-Large is ~340M params

Rank	Method	Test MRR	Validation MRR	Contact	References	#Params	Hardware	Date
1	PairRE (200dim)	0.5208 \pm 0.0027	0.5423 \pm 0.0020	Linlin Chao	Paper , Code	500,334,800	Tesla P100 (16GB GPU)	Jan 28, 2021
2	RotatE (250dim)	0.4332 \pm 0.0025	0.4353 \pm 0.0028	Hongyu Ren – OGB team	Paper , Code	1,250,435,750	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021
3	TransE (500dim)	0.4256 \pm 0.0030	0.4272 \pm 0.0030	Hongyu Ren – OGB team	Paper , Code	1,250,569,500	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021
4	ComplEx (250dim)	0.4027 \pm 0.0027	0.3759 \pm 0.0016	Hongyu Ren – OGB team	Paper , Code	1,250,569,500	Quadro RTX 8000 (45GB GPU)	Jan 23, 2021

BERT (340M params) - disruption in NLP ✓
KG embs (>1B params) - 🙄

Life beyond shallow embedding?

Do we really need to learn & store the whole **shallow** embedding matrix $|E| \times \dim$?

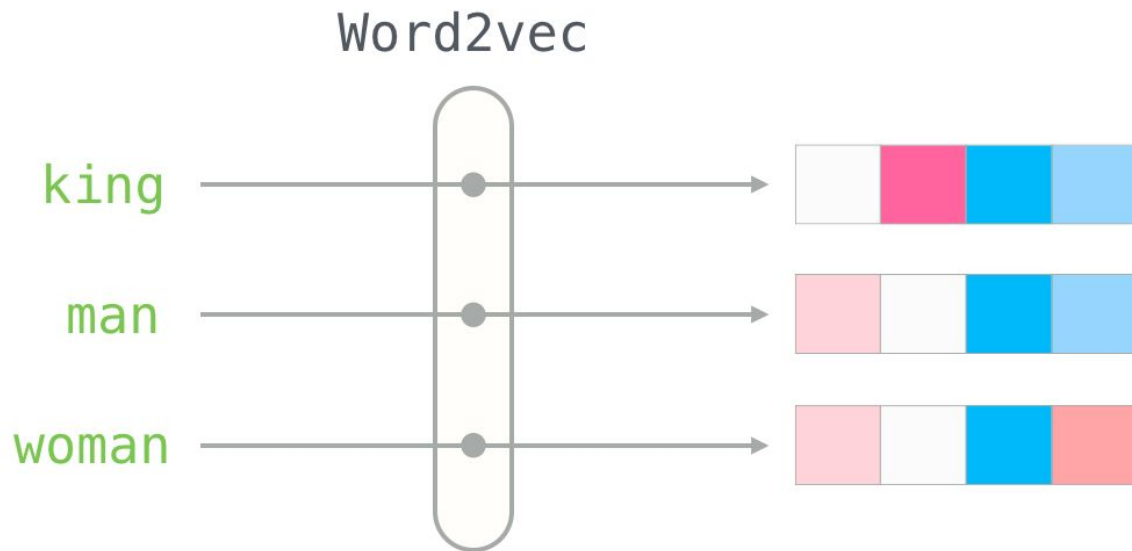
Trying to fit a 100M x 200 tensor on a Tesla V100 ->



Plan

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Back to 2014



Unseen words = [OOV] (out-of-vocabulary)

Byte-Pair Encoding / WordPiece

Dictionary

5 l o w
2 l o w e r
6 n e w **est**
3 w i d **est**

Vocabulary

l, o, w, e, r, n, w, s, t, i, d, es, **est**

Add a pair (es, t) with freq 9

Byte-Pair Encoding / WordPiece

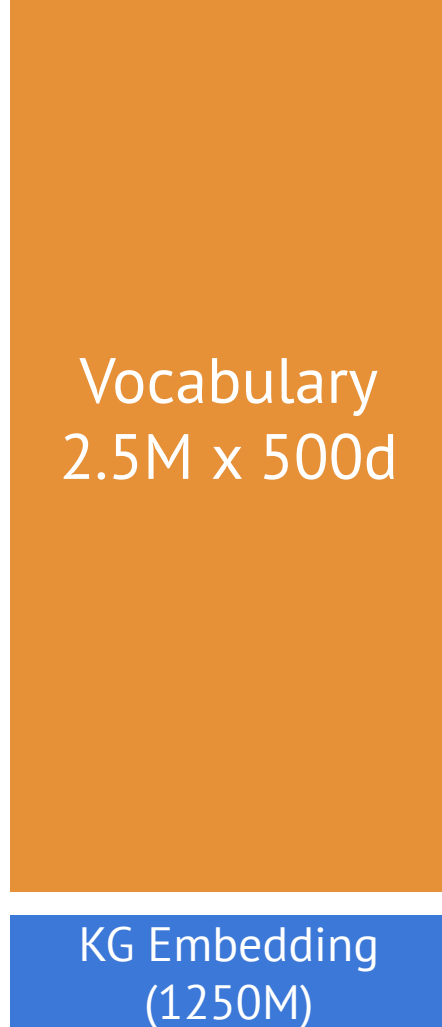
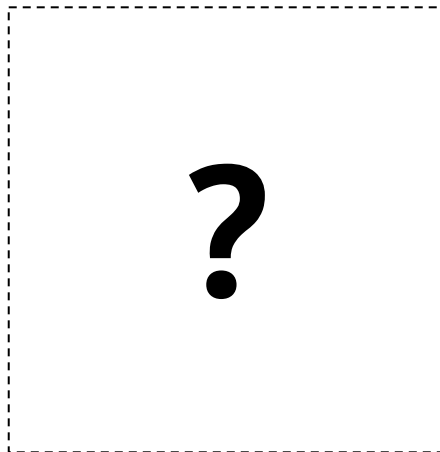
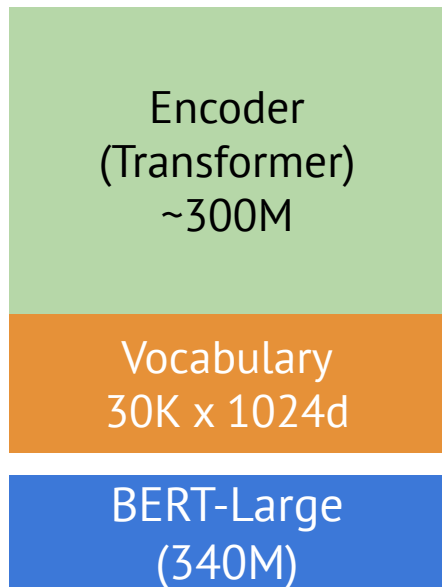
"I love tacos, apples, and tea!"

i love tacos , app ##les , and t ##e ##a !

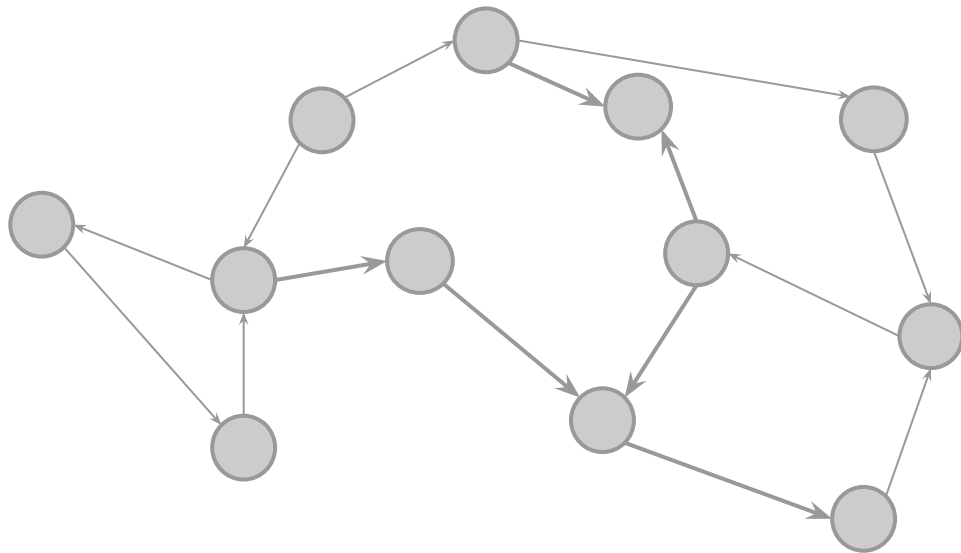
6 7 8 5 10 11 5 9 30 41 37 3

- Fixed-size vocab of subword units (30-50K)
- We can tokenize any unseen word

Tokenizing KGs

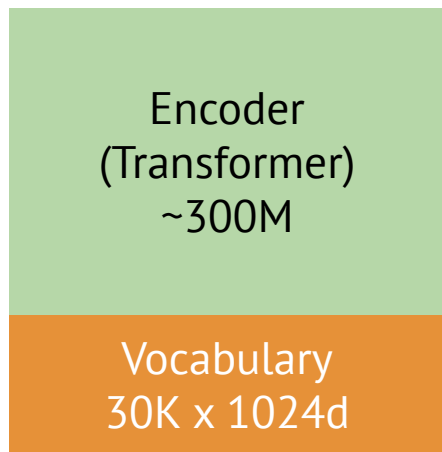


Tokenization + Graphs?

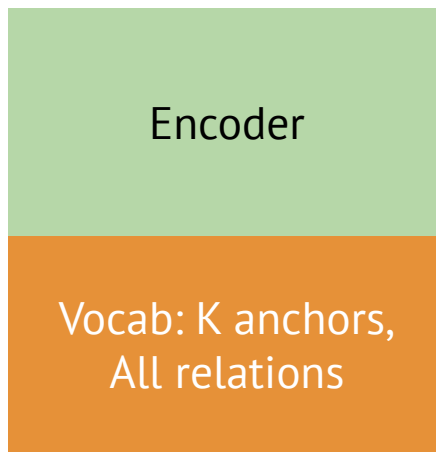


If nodes in a graph are
"words",
can we design a
fixed-size vocab of
"sub-word" units?

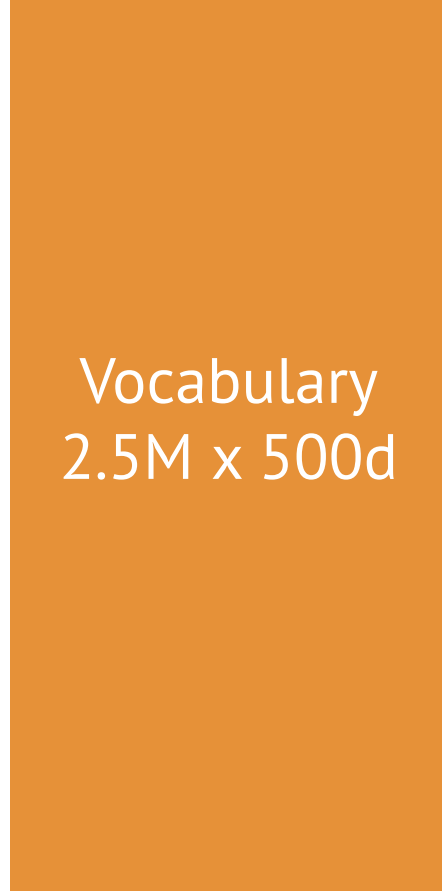
Tokenizing KGs



BERT-Large
(340M)



NodePiece



KG Embedding
(1250M)

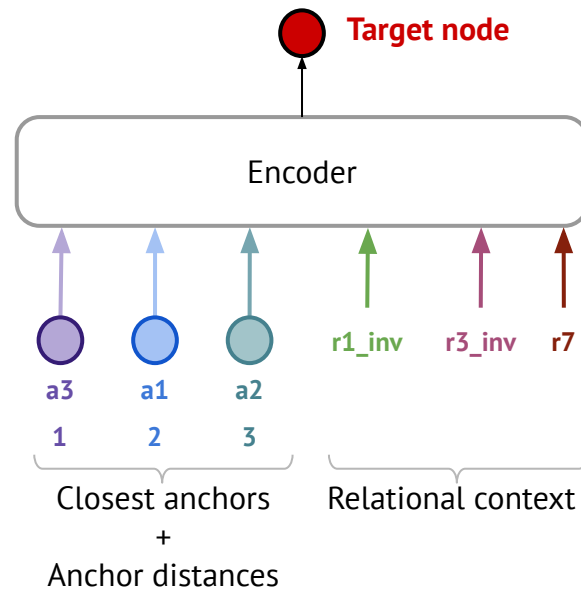
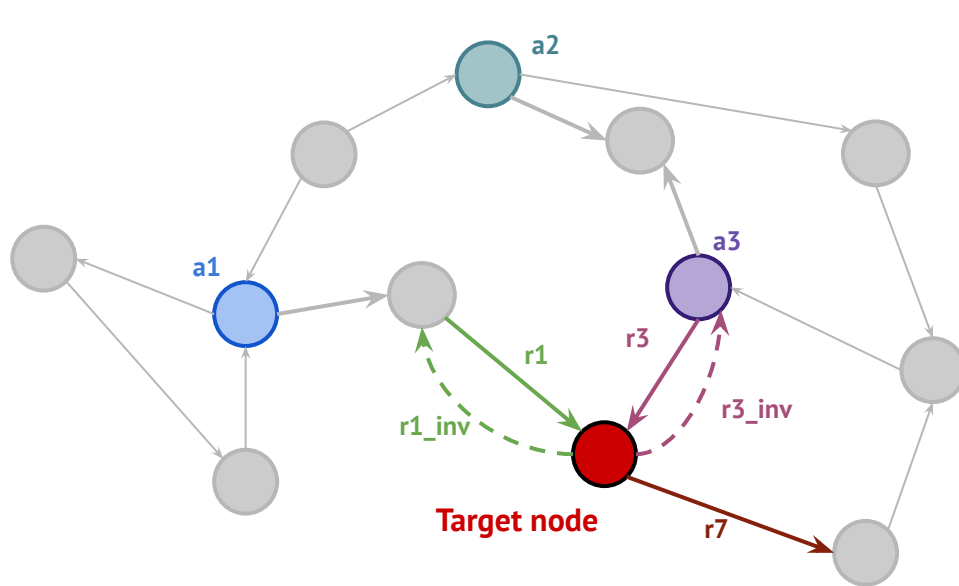
Tokenizing KGs

Shallow embedding, only
known words, otherwise
OOV

Compositional
representations,
subword units

Language	Word2vec, GloVe	Byte-Pair Encoding, WordPiece
Graphs	All KG embedding algorithms (TransE, etc)	NodePiece

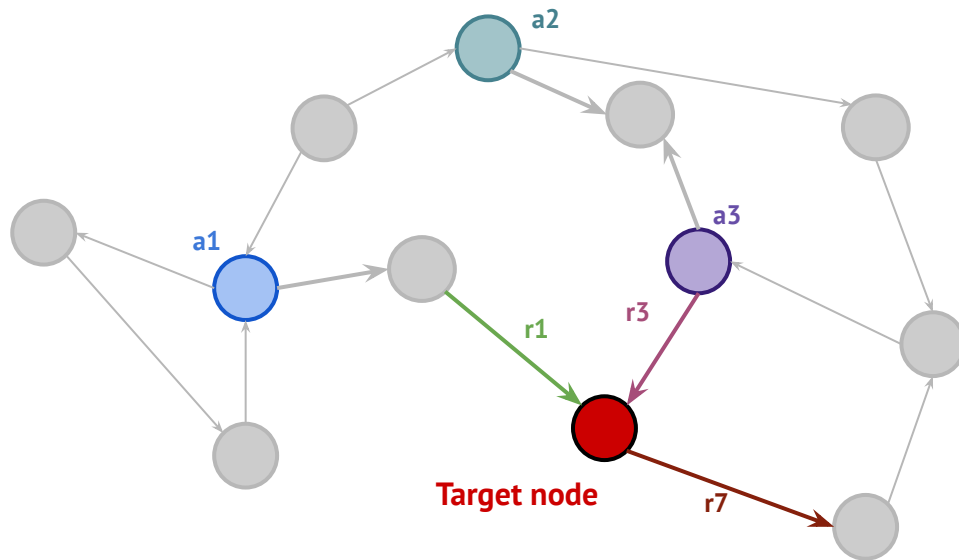
NodePiece - “*subword units*” for KGs



Vocabulary = Anchors + Relation types

Inductive out-of-the-box: unseen nodes are “tokenized” with the same Vocab

Tokenization Idea

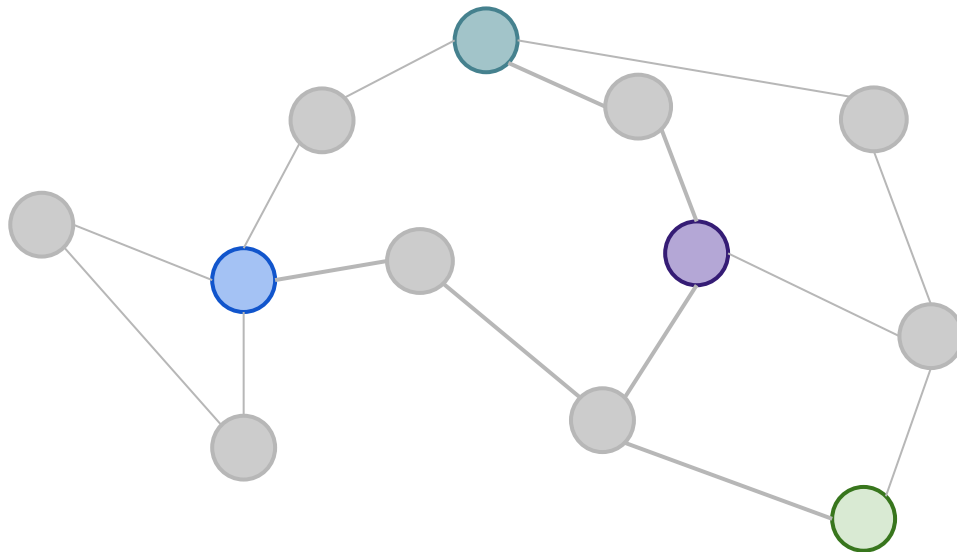


Represent an entity e as
a set of **k most similar** tokens t

$$\max \text{sim}(e, \{t_i\}_{i \in k})$$

- Basic case: similarity as shortest path distance
- Can be generalized to non-Euclidean spaces

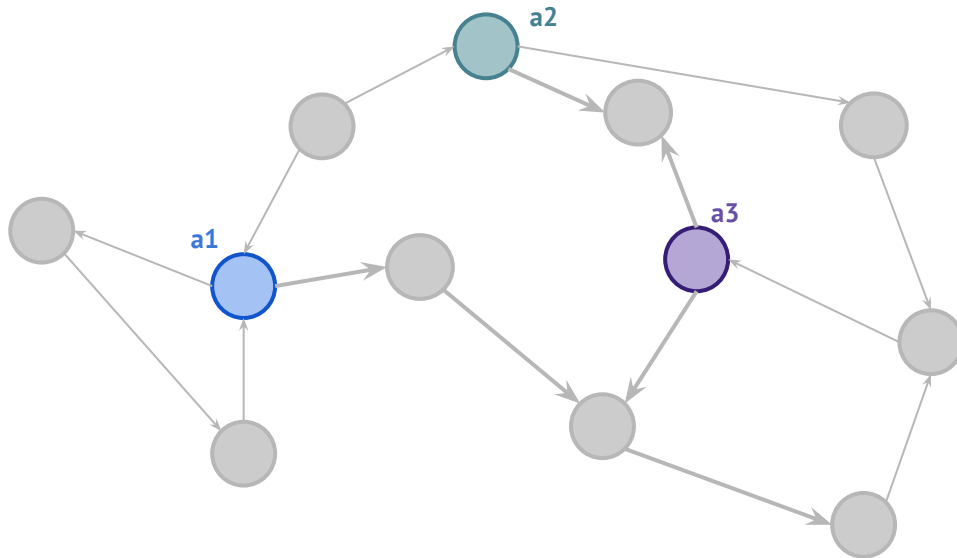
Anchor Node Selection



Ideal: Anchors = Dominating Set

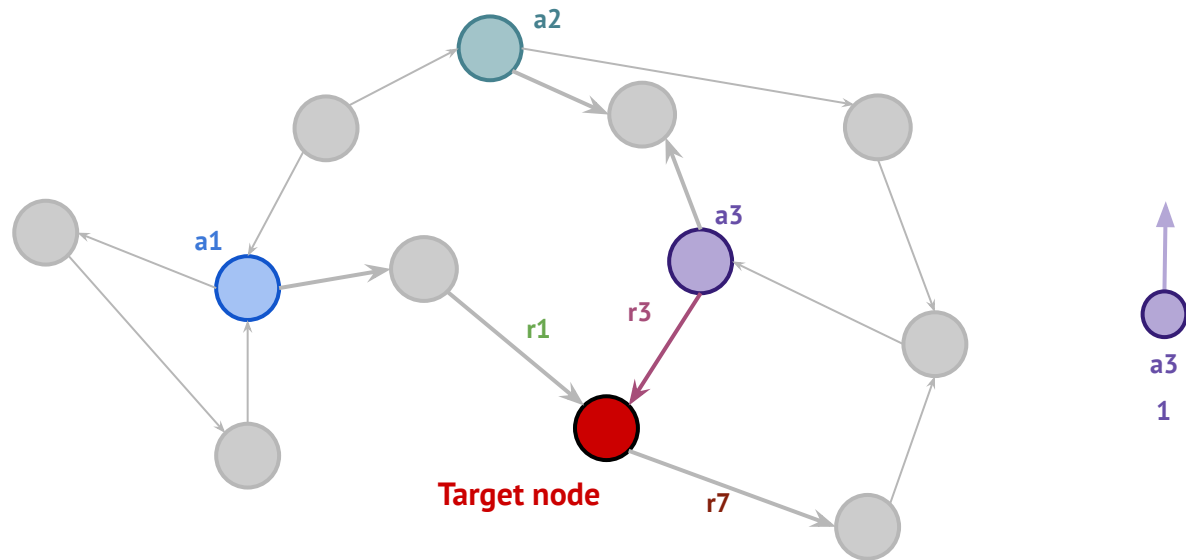
- ✓ Minimized distances
- 😓 NP complete
- 😓 Even k-hop Dominating Set is NP complete

Anchor Node Selection



Current strategy:
40% top degrees
40% top PPR
20% random

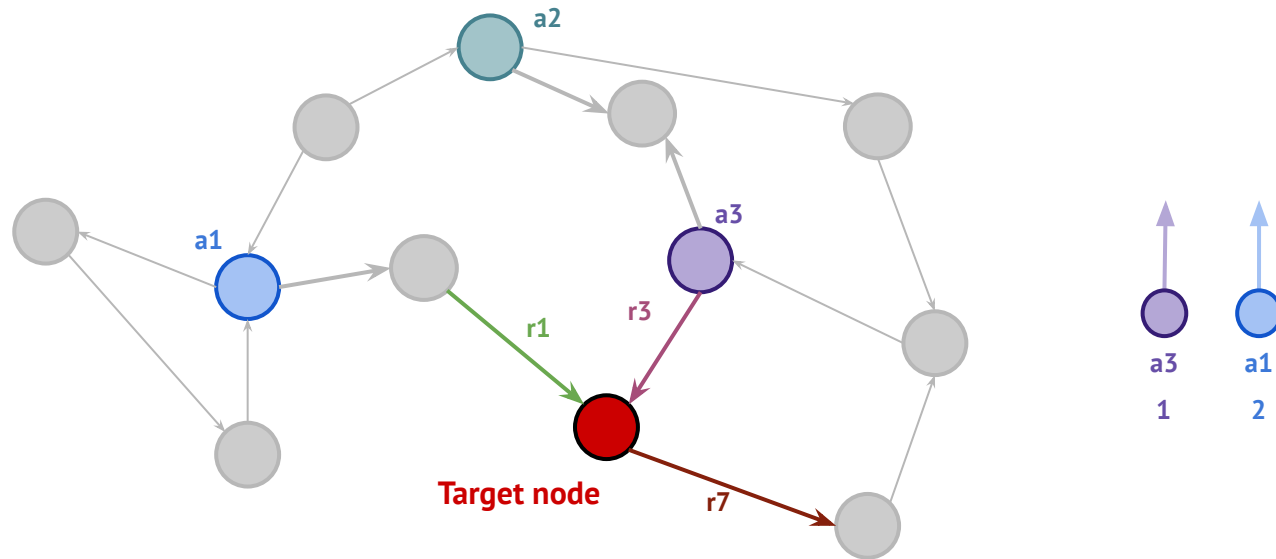
Tokenization



BFS from the target node until we reach $|K|$ anchors

- Can be done in forward pass
- Can be pre-processed and saved

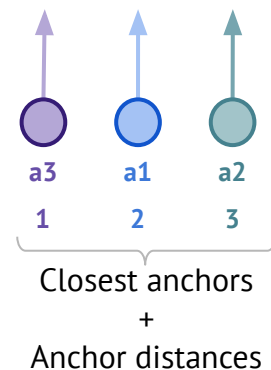
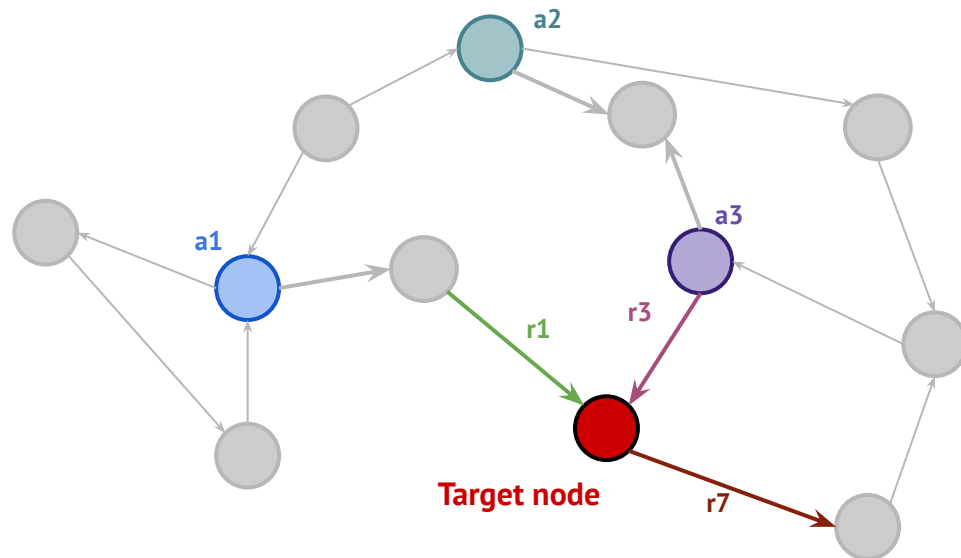
Tokenization



BFS from the target node until we reach $|K|$ anchors

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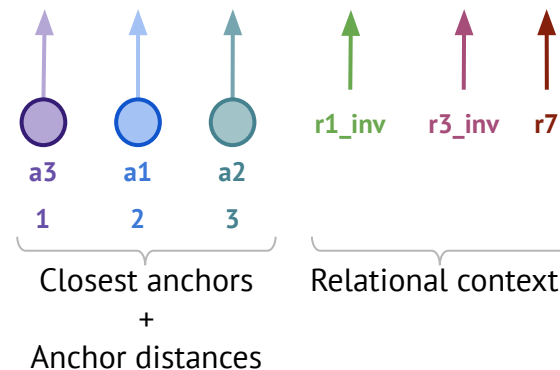
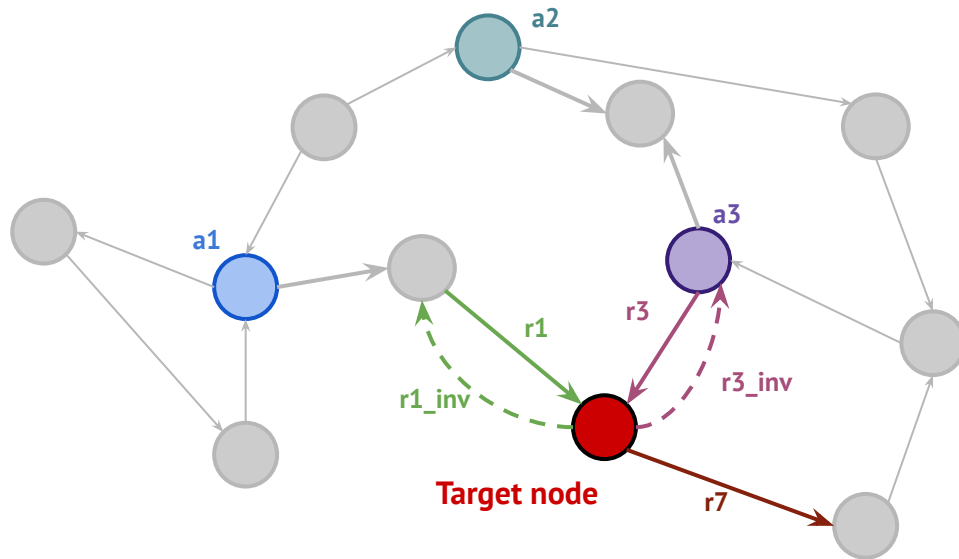
Tokenization



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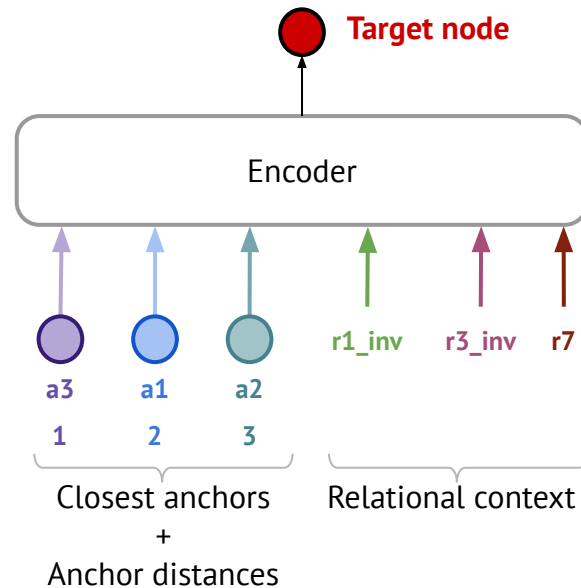
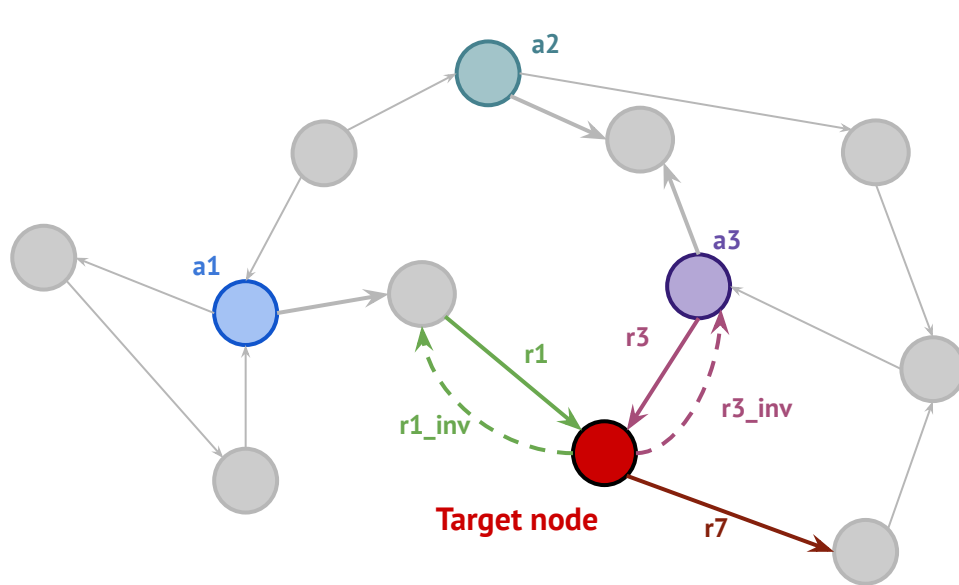
Tokenization



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Tokenization



BFS from the target node until we reach $|K|$ anchors

- Can be done in forward pass
- Can be pre-processed and saved

Tokenizing Einstein

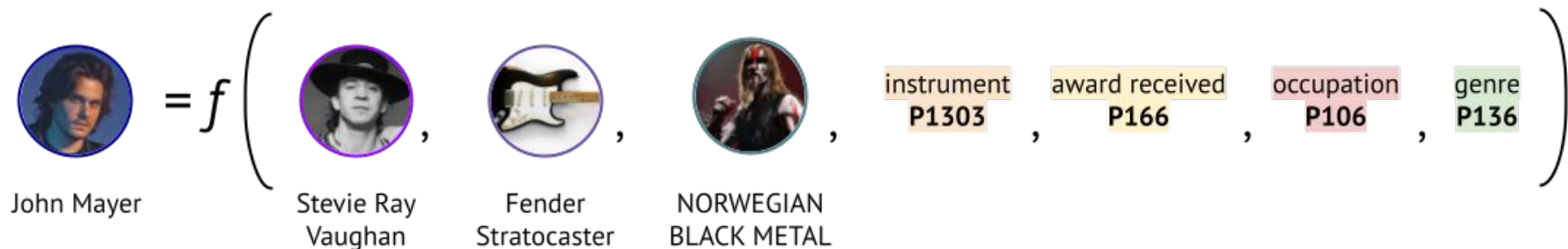


3 nearest anchors

4 unique outgoing relations in the context



Tokenizing John Mayer

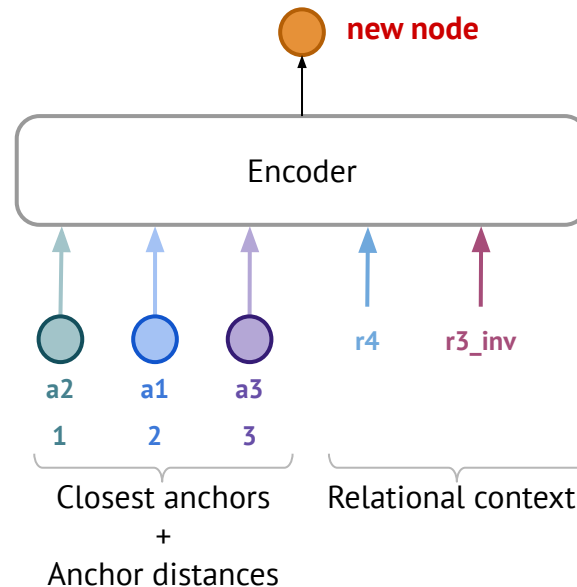
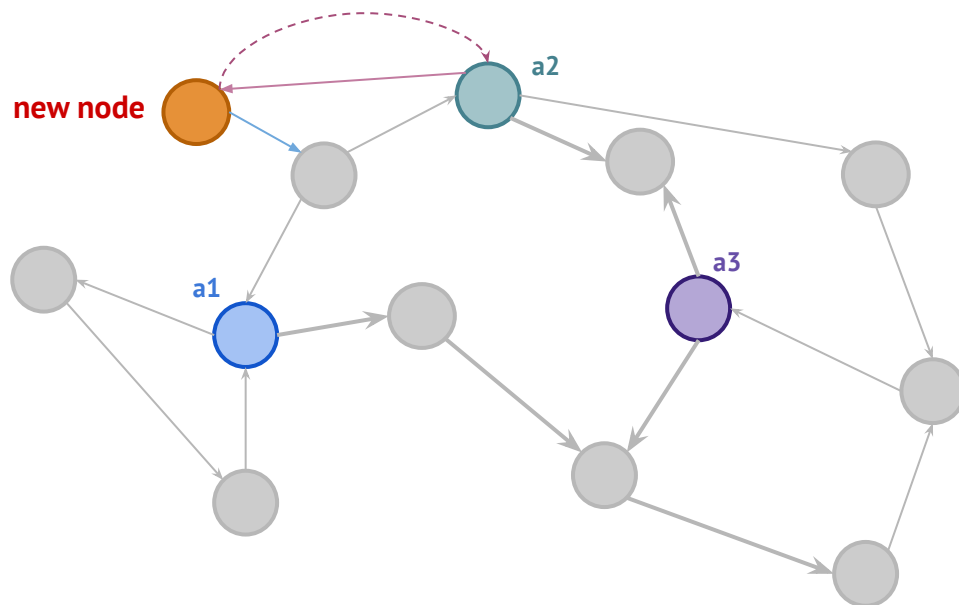


3 nearest anchors

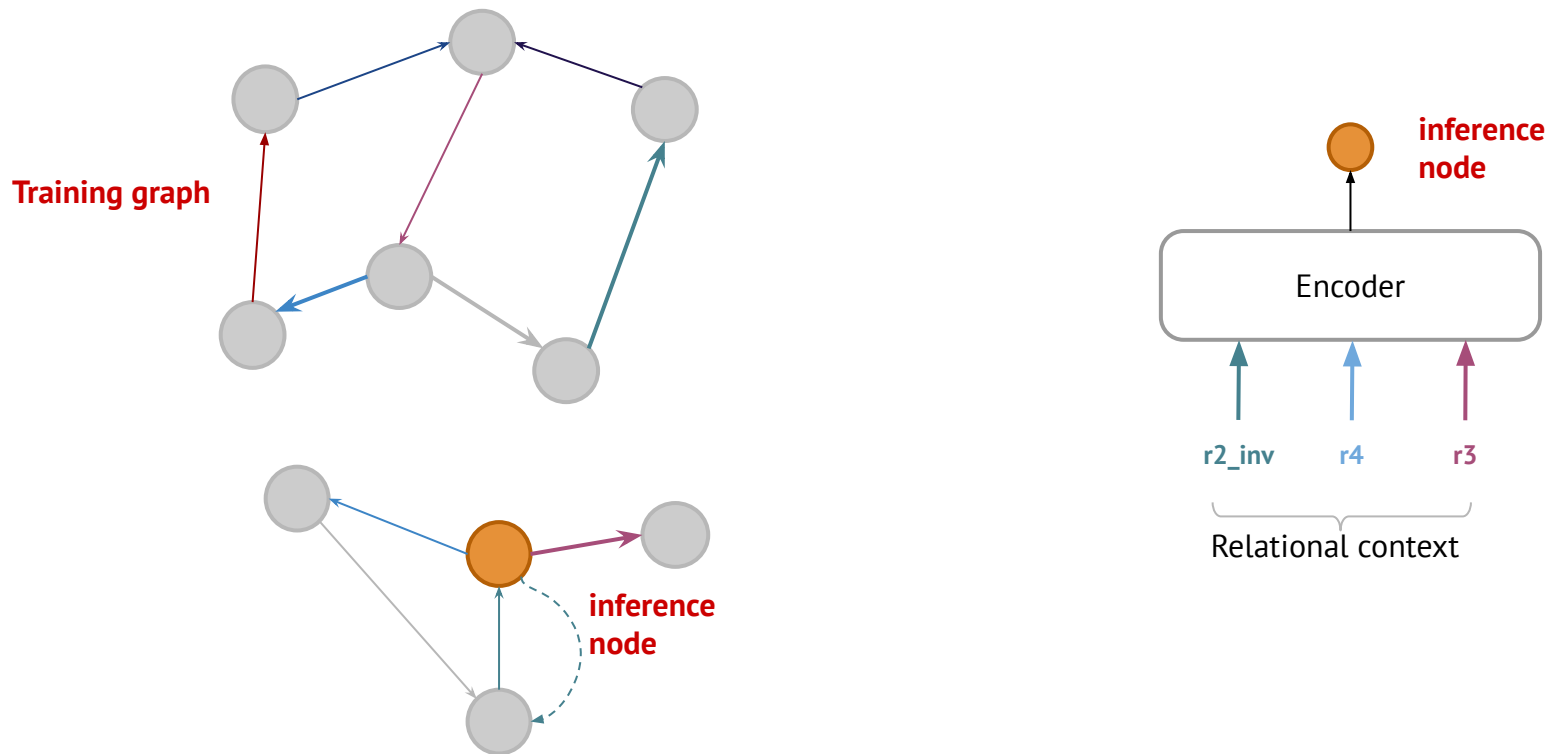
4 unique outgoing relations in the context



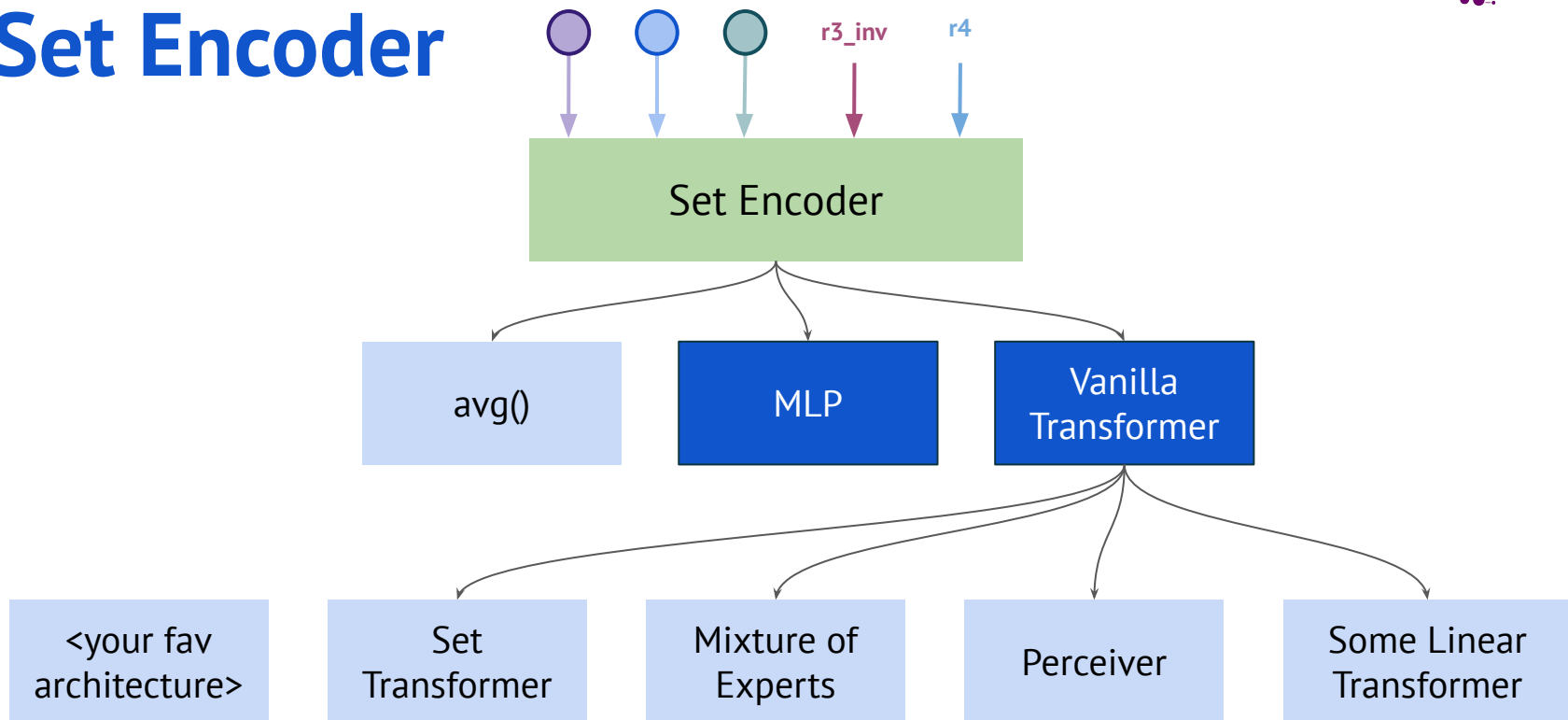
Unseen Node Tokenization



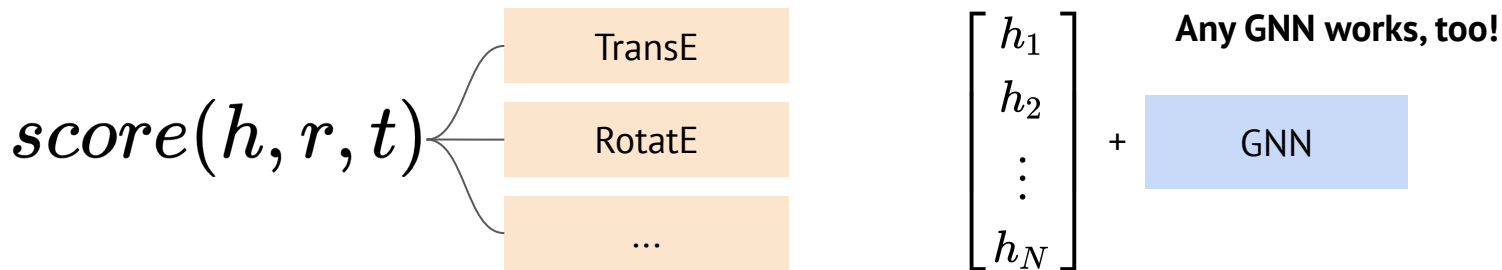
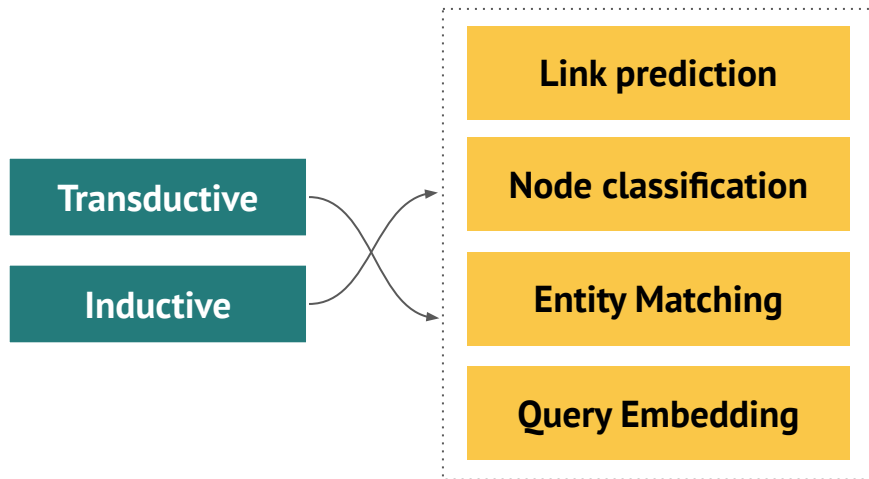
Inductive Node Tokenization



Set Encoder



New Downstream Tasks



Transductive Link Prediction

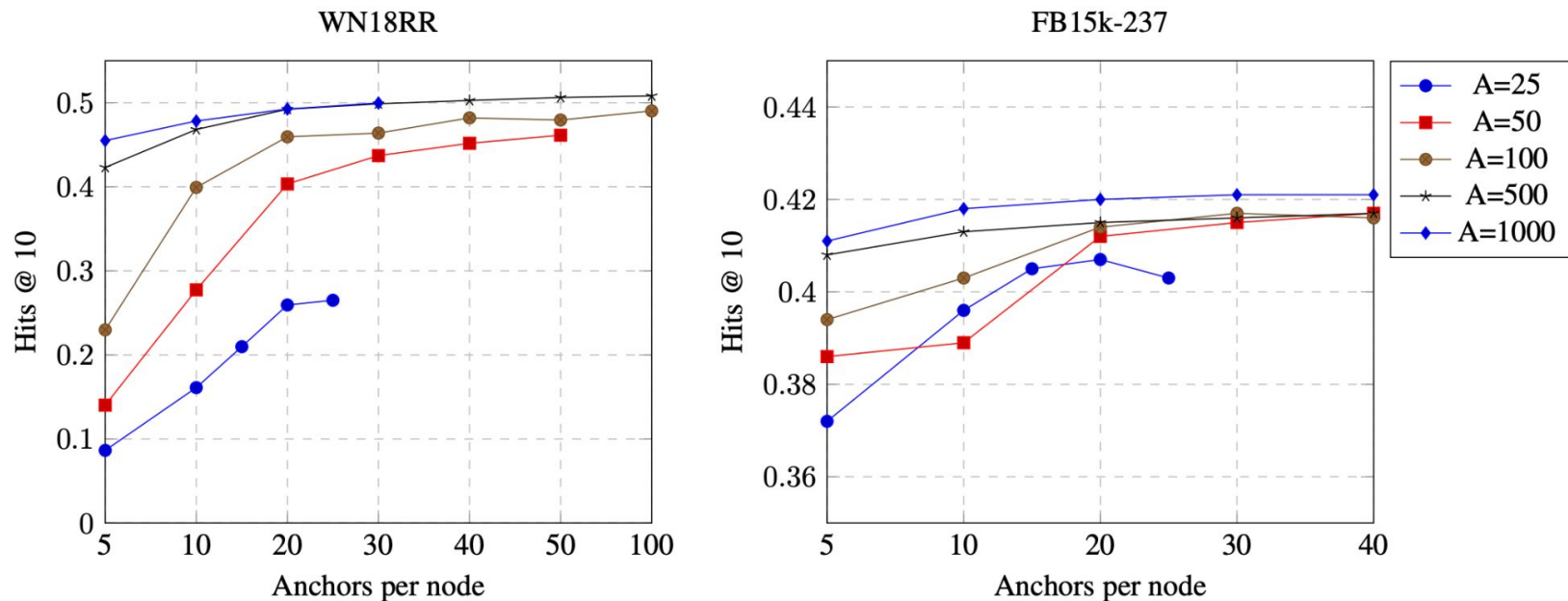


Figure 2: Combinations of total anchors A and anchors per node. Denser FB15k-237 saturates faster on smaller A while sparse WN18RR saturates at around 500 anchors.

NodePiece Experiments: Summary



10x fewer parameters while retaining **90%** of transductive LP



2x better compared to shallow models of similar #params



Relation Prediction and Node Classification: no anchors is better!



Inductive out-of-the-box and very competitive

Table 6: Node classification results. $|V|$ denotes vocabulary size (anchors + relations), #P is a total parameter count (millions).

	$ V $	#P (M)	WD50K (5% labeled)			WD50K (10% labeled)		
			ROC-AUC	PRC-AUC	Hard Acc	ROC-AUC	PRC-AUC	Hard Acc
MLP	46k + 1k	4.1	0.503	0.016	0.001	0.510	0.017	0.002
CompGCN	46k + 1k	4.4	0.836	0.280	0.176	0.834	0.265	0.161
NodePiece + GNN	50 + 1k	0.75	0.981	0.443	0.513	0.981	0.450	0.516
- no rel. context	50 + 1k	0.64	0.982	0.446	0.534	0.982	0.449	0.530
- no distances	50 + 1k	0.74	0.981	0.448	0.516	0.981	0.448	0.513
- no anchors, rels only	0 + 1k	0.54	0.984	0.453	0.532	0.984	0.456	0.533



**Yesterday this slide
had a UMAP
visualization**

OGB WikiKG 2 : NodePiece is New SOTA

Leaderboard for [ogbl-wikikg2](#)

The MRR score on the test and validation sets. The higher, the better.

Package: $\geq 1.2.4$

Deprecated [ogbl-wikikg](#) leaderboard can be found [here](#).

July 2022

Rank	Method	Ext. data	Test MRR	Validation MRR	Contact	References	#Params	Hardware	Date
1	StarGraph + TripleRE	No	0.7201 \pm 0.0011	0.7288 \pm 0.0008	Hongzhu Li (360AI)	Paper , Code	86,762,146	Tesla A100(40GB)	May 30, 2022
2	Trans	No	0.6939 \pm 0.0011	0.7058 \pm 0.0018	Xuanyu Zhang (DXM AI)	Paper , Code	38,430,804	Tesla V100 (16GB)	Apr 19, 2022
3	Trans	No	0.6882 \pm 0.0019	0.6988 \pm 0.0006	Xuanyu Zhang (DXM AI)	Paper , Code	19,215,402	Tesla V100 (16GB)	Apr 28, 2022
4	TripleRE + NodePiece	No	0.6866 \pm 0.0014	0.6955 \pm 0.0008	Long Yu (360AI)	Paper , Code	36,421,802	Tesla A100(40GB)	Feb 24, 2022
5	InterHT	No	0.6779 \pm 0.0018	0.6893 \pm 0.0015	Baixin Wang (HFL)	Paper , Code	19,215,402	Tesla V100 (32GB)	Feb 10, 2022
6	TripleRE + NodePiece	No	0.6582 \pm 0.0020	0.6616 \pm 0.0018	Long Yu (360AI)	Paper , Code	7,289,002	Tesla A100(40GB)	Dec 25, 2021
7	ComplEx-RP (50dim)	No	0.6392 \pm 0.0045	0.6561 \pm 0.0070	Yihong Chen (UCL NLP & FAIR London)	Paper , Code	250,167,400	Tesla V100 (32GB)	Nov 23, 2021

NodePiece-enabled models

OGB WikiKG 2

Input graph: 2.5M nodes, 16M edges, ~1K edge types

- **20K** anchors (< **1%** total nodes) -> 4M params
- 0 anchors / 0 node embeddings -> 0.476 MRR
- No relations in node hashes -> also OK
- “Word length” - 32 tokens
 - 20 anchors per node
 - 12 relations in context

Table 4: Test MRR and parameter budget on OGB WikiKG 2.

Model	#Params	MRR
NP + AutoSF	6.9M	0.570 \pm 0.003
- rel. context	5.9M	0.592 \pm 0.003
- anc. dists	6.9M	0.570 \pm 0.004
- no anchors	1.3M	0.476 \pm 0.001
AutoSF	500M	0.546 \pm 0.005
PairRE	500M	0.521 \pm 0.003
RotatE	1250M	0.433 \pm 0.002
TransE	1250M	0.426 \pm 0.003

Inductive Link Prediction

Inference graphs are disjoint with training (new nodes)

NodePiece + CompGCN encoder = SOTA on many tasks on relation-rich graphs

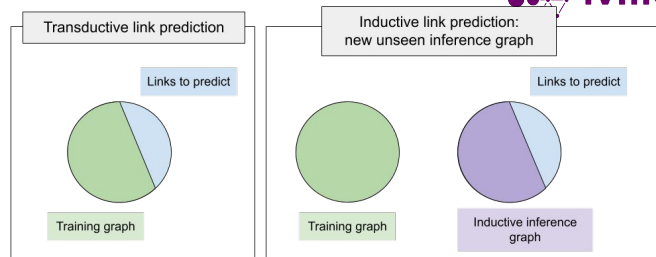


Table 5: Inductive link prediction results, Hits@10. Best results are in **bold**, second best are underlined. † results taken from Teru et al. (2020). NBFNet results taken from Zhu et al. (2021).

Class	Method	FB15k-237				WN18RR				NELL-995			
		V1	V2	V3	V4	V1	V2	V3	V4	V1	V2	V3	V4
Path	Neural LP †	0.529	0.589	0.529	0.559	0.744	0.689	0.462	0.671	0.408	0.787	0.827	0.806
	DRUM †	0.529	0.587	0.529	0.559	0.744	0.689	0.462	0.671	0.194	0.786	0.827	0.806
	RuleN †	0.498	0.778	0.877	0.856	0.809	0.782	0.534	0.716	0.535	0.818	0.773	0.614
GNN	GraIL †	0.642	0.818	0.828	0.893	0.825	0.787	0.584	0.734	<u>0.595</u>	0.933	<u>0.914</u>	0.732
	NBFNet	<u>0.834</u>	0.949	0.951	0.960	0.948	0.905	0.893	0.890	-	-	-	-
	NP + CompGCN	0.873	0.939	0.944	0.949	<u>0.830</u>	0.886	<u>0.785</u>	<u>0.807</u>	0.890	<u>0.901</u>	0.936	0.893

Plan

- Graph Reasoning Tasks
- Featurization via Tokenization: NodePiece
- **Featurization via Labeling Trick:
Neural Bellman-Ford and GNN-QE**
- Past, Today, Future

The Labeling Trick

Idea: for **each** link we predict, **instantiate** a graph with **unique** initial node labels (**features**)

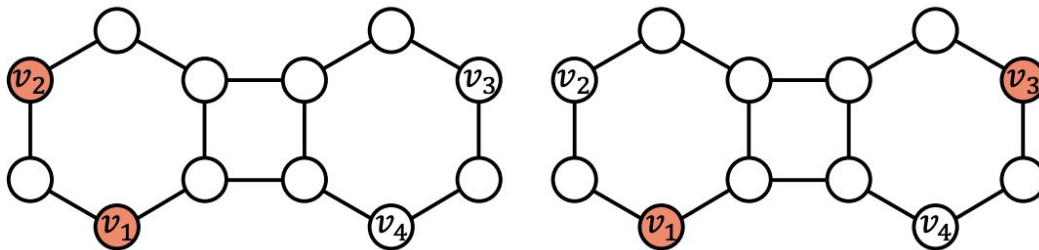


Figure 2: When we predict (v_1, v_2) , we will label these two nodes differently from the rest, so that a GNN is aware of the target link when learning v_1 and v_2 's representations. Similarly, when predicting (v_1, v_3) , nodes v_1 and v_3 will be labeled differently. This way, the representation of v_2 in the left graph will be different from that of v_3 in the right graph, enabling GNNs to distinguish the non-isomorphic links (v_1, v_2) and (v_1, v_3) .

The Labeling Trick

Idea: for **each** link we predict, **instantiate** a graph with **unique** initial node labels (**features**)

- SEAL (homogeneous link prediction, still SOTA on OGB)
- GraIL (KG link prediction, first inductive method)
- Neural Bellman-Ford (homogeneous + KG link prediction, current SOTA)

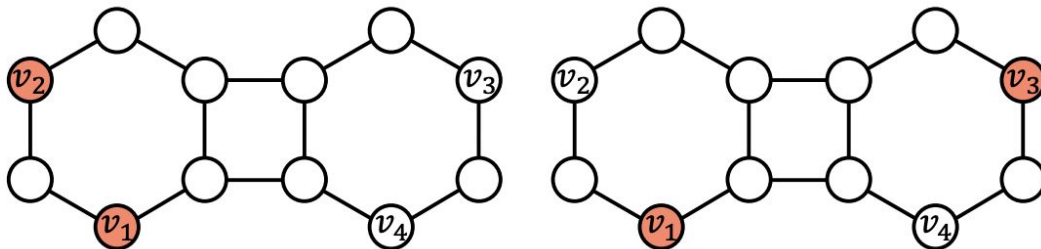
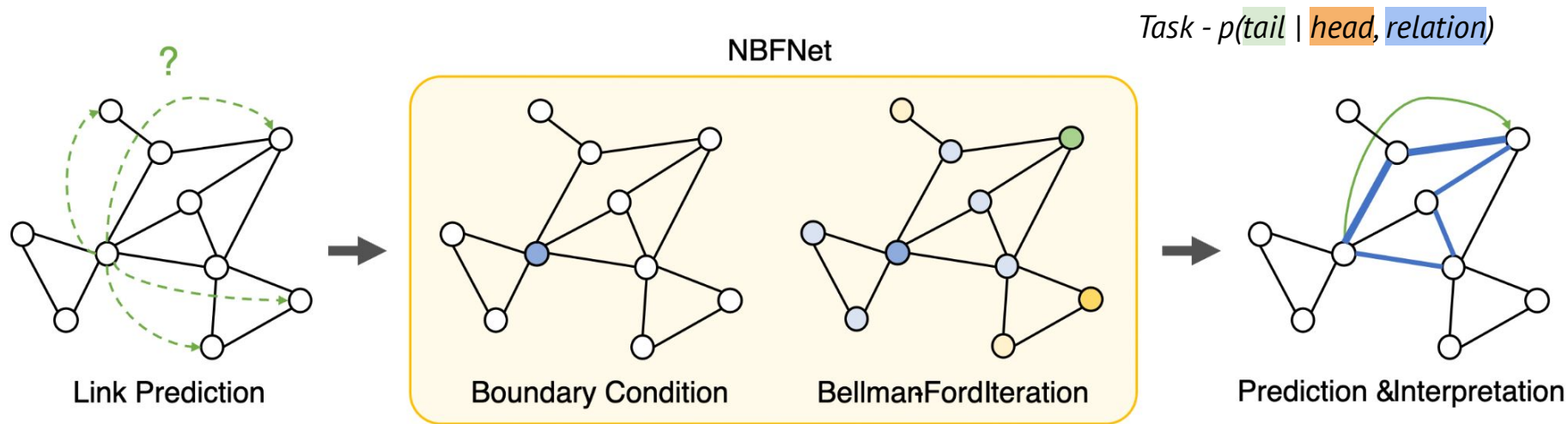


Figure 2: When we predict (v_1, v_2) , we will label these two nodes differently from the rest, so that a GNN is aware of the target link when learning v_1 and v_2 's representations. Similarly, when predicting (v_1, v_3) , nodes v_1 and v_3 will be labeled differently. This way, the representation of v_2 in the left graph will be different from that of v_3 in the right graph, enabling GNNs to distinguish the non-isomorphic links (v_1, v_2) and (v_1, v_3) .

Neural Bellman-Ford



Idea:

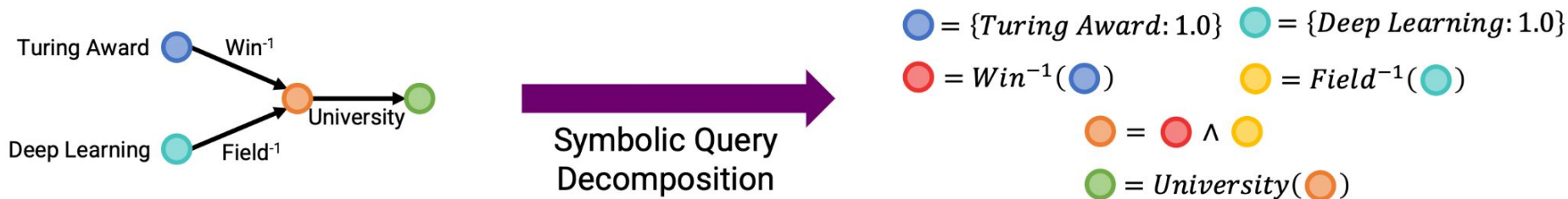
1. Relations do not change at inference -> we can learn relation (edge type) embeddings
2. Initialize **head node feature** with the learnable **relation** vector (**query**)
3. Propagate for L layers, take final representations as final node features

Neural Bellman-Ford

Table 4: Homogeneous graph link prediction results. Results of VGAE and S-VGAE are taken from their original papers [32, 12].

Class	Method	Cora		Citeseer		PubMed	
		AUROC	AP	AUROC	AP	AUROC	AP
Path-based	Katz Index [30]	0.834	0.889	0.768	0.810	0.757	0.856
	Personalized PageRank [42]	0.845	0.899	0.762	0.814	0.763	0.860
	SimRank [28]	0.838	0.888	0.755	0.805	0.743	0.829
Embeddings	DeepWalk [43]	0.831	0.850	0.805	0.836	0.844	0.841
	LINE [53]	0.844	0.876	0.791	0.826	0.849	0.888
	node2vec [17]	0.872	0.879	0.838	0.868	0.891	0.914
GNNs	VGAE [32]	0.914	0.926	0.908	0.920	0.944	0.947
	S-VGAE [12]	0.941	0.941	0.947	0.952	0.960	0.960
	SEAL [73]	0.933	0.942	0.905	0.924	0.978	0.979
	TLC-GNN [67]	0.934	0.931	0.909	0.916	0.970	0.968
	NBFNet	0.956	0.962	0.923	0.936	0.983	0.982

GNN-QE: NBFNet + Multi-hop Reasoning

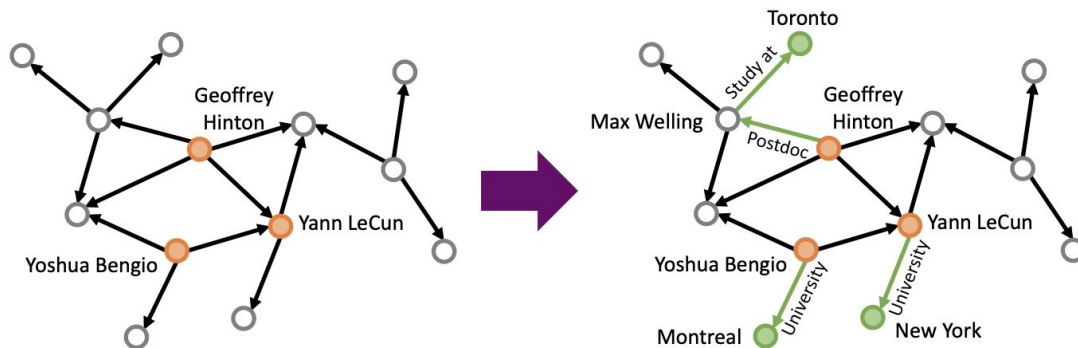


Each variable is a fuzzy set of entities, where each element in the set has a probability.

- Each relation projection (simple link prediction) step is modelled by a L-layer NBFNet
- NBFNet returns a probability distribution (scalars) over all entities (fuzzy set)

GNN-QE: NBFNet + Multi-hop Reasoning

Neural Relation Projection $University(\text{orange circle})$

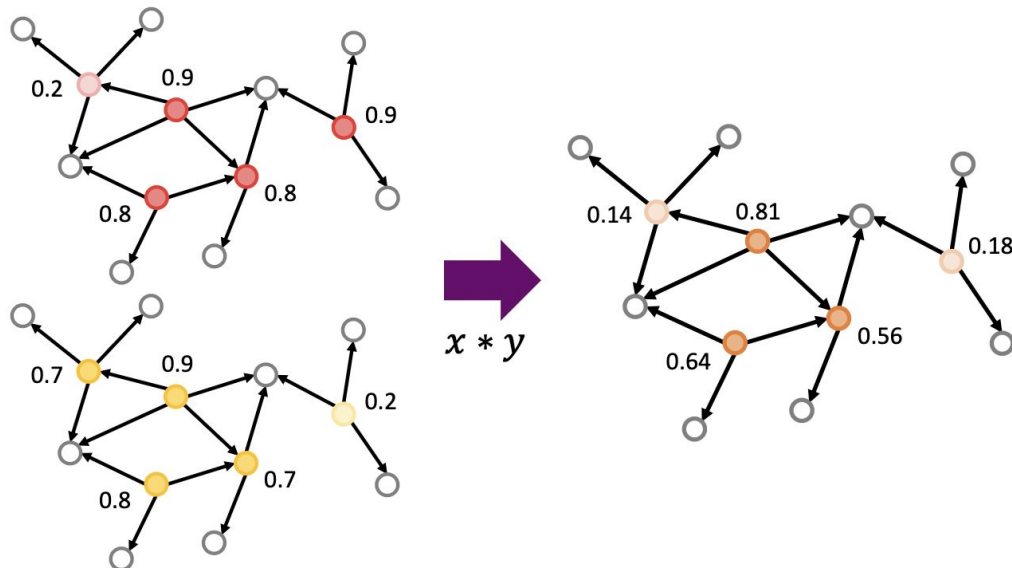


Use a **GNN** to propagate the input fuzzy set and get the output fuzzy set.

The relation *University* is used to guide the propagation towards paths that can predict the relation *University*.

GNN-QE: NBFNet + Multi-hop Reasoning

Fuzzy Logic Operations



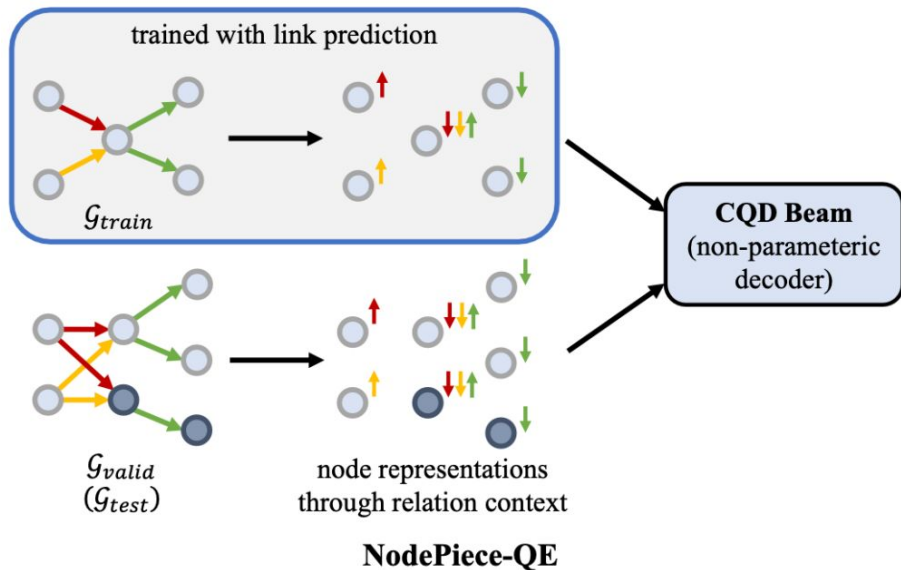
Use **product fuzzy logic** to model logic operations (e.g. \neg , \wedge , \vee) over fuzzy sets.

1. Logical operators as algebraic operations
2. Resulting probability **distribution** is used as **scalar weights** for the next hop graph initialization

The Essence of Inductiveness

What is **invariant** in inductive reasoning setups? **Relation types**

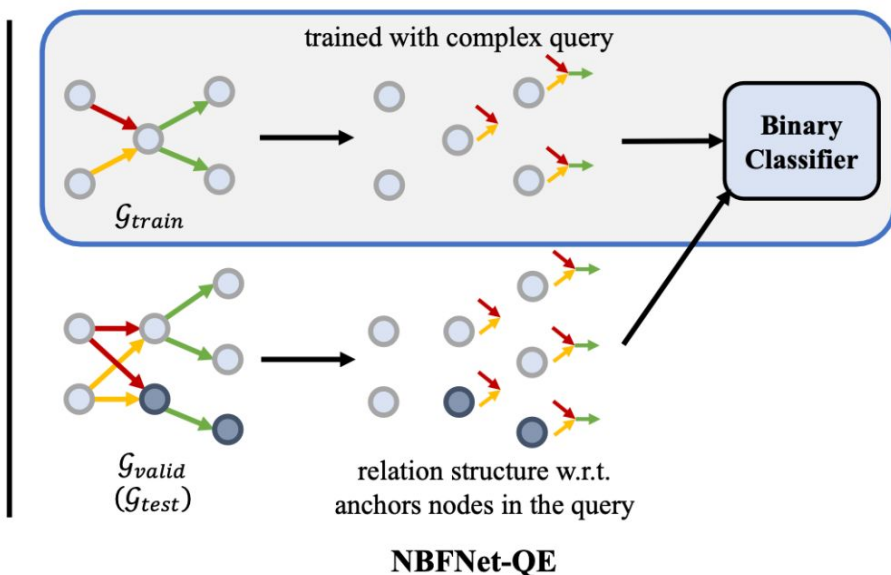
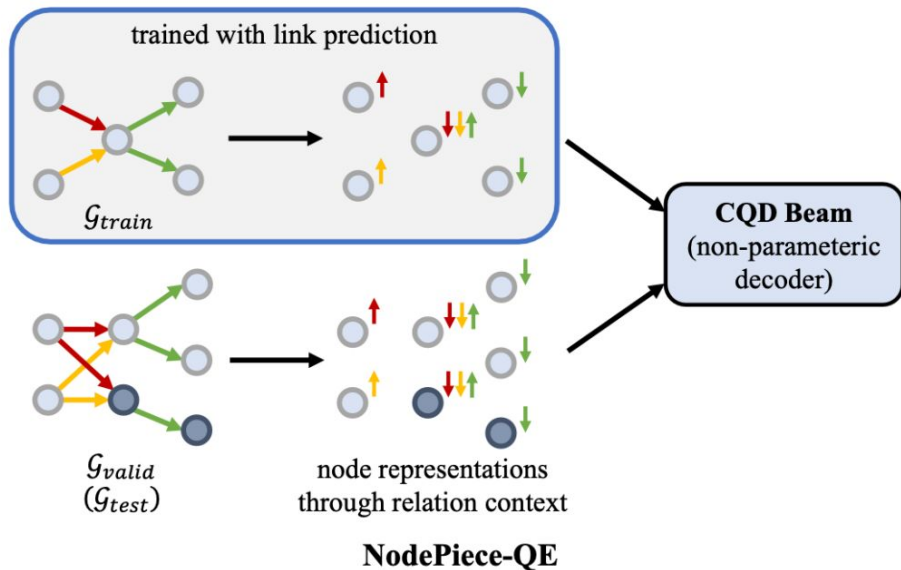
- NodePiece - parameterization through relational context



The Essence of Inductiveness

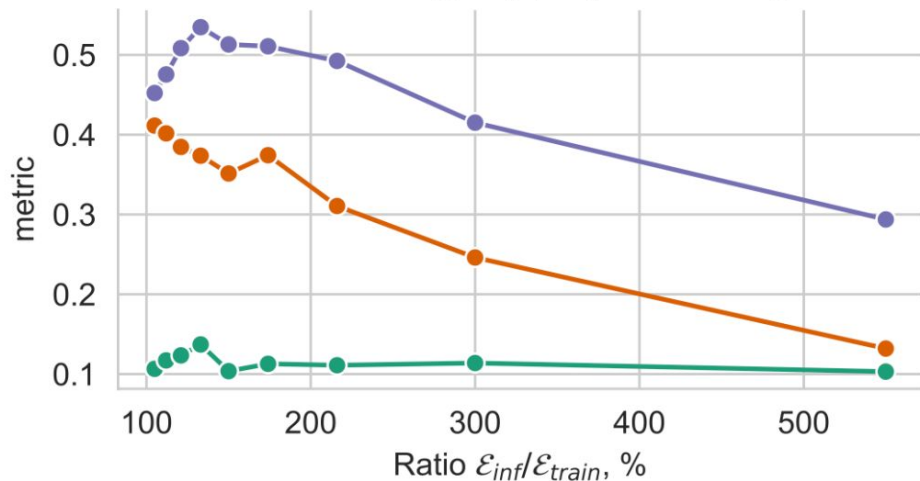
What is **invariant** in inductive reasoning setups? **Relation types**

- NodePiece - parameterization through relational context
- GNN-QE - parameterization through relational structure

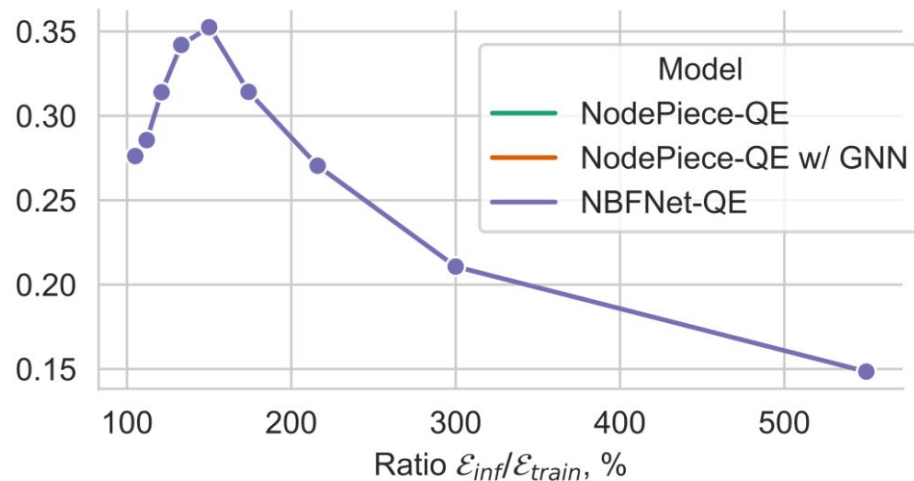


Inductive Generalization to Larger Test Graphs is Still a Problem

Metric = Hits@10 | query = EPFO Avg



Metric = Hits@10 | query = neg avg



Plan

- Graph Reasoning Tasks
- Featurization via Tokenization: NodePiece
- Featurization via Labeling Trick:
Neural Bellman-Ford and GNN-QE
- **Past, Today, Future**

Space of KG Tasks in 2019

Transductive

Triples

SETTING

TASK

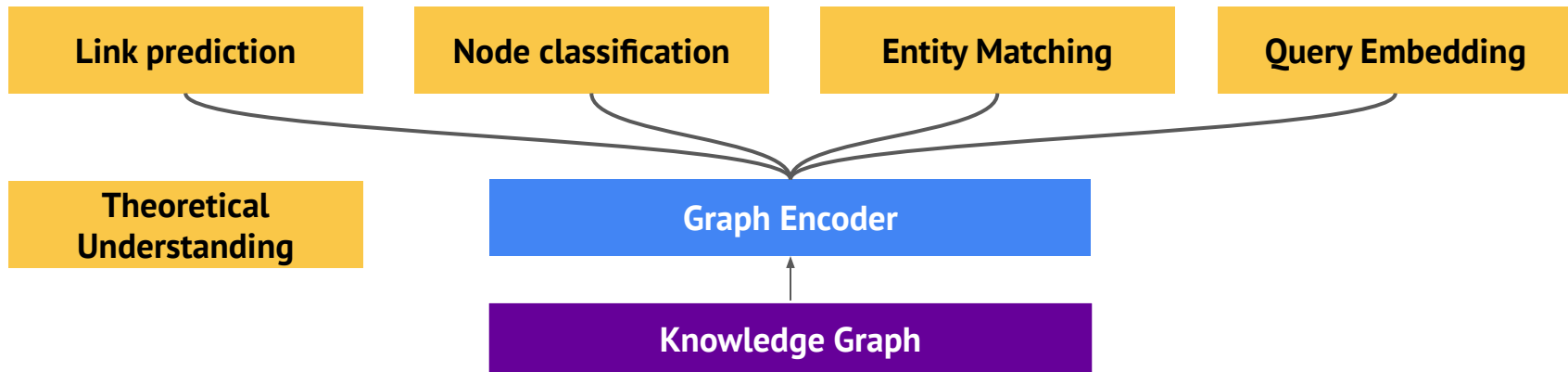
Link prediction

Space of KG Tasks Today

Transductive	Triples	Supervised	Unimodal	Small
Inductive	Hyper-relational	Unsupervised	Multimodal	Large (sampling)

SETTING

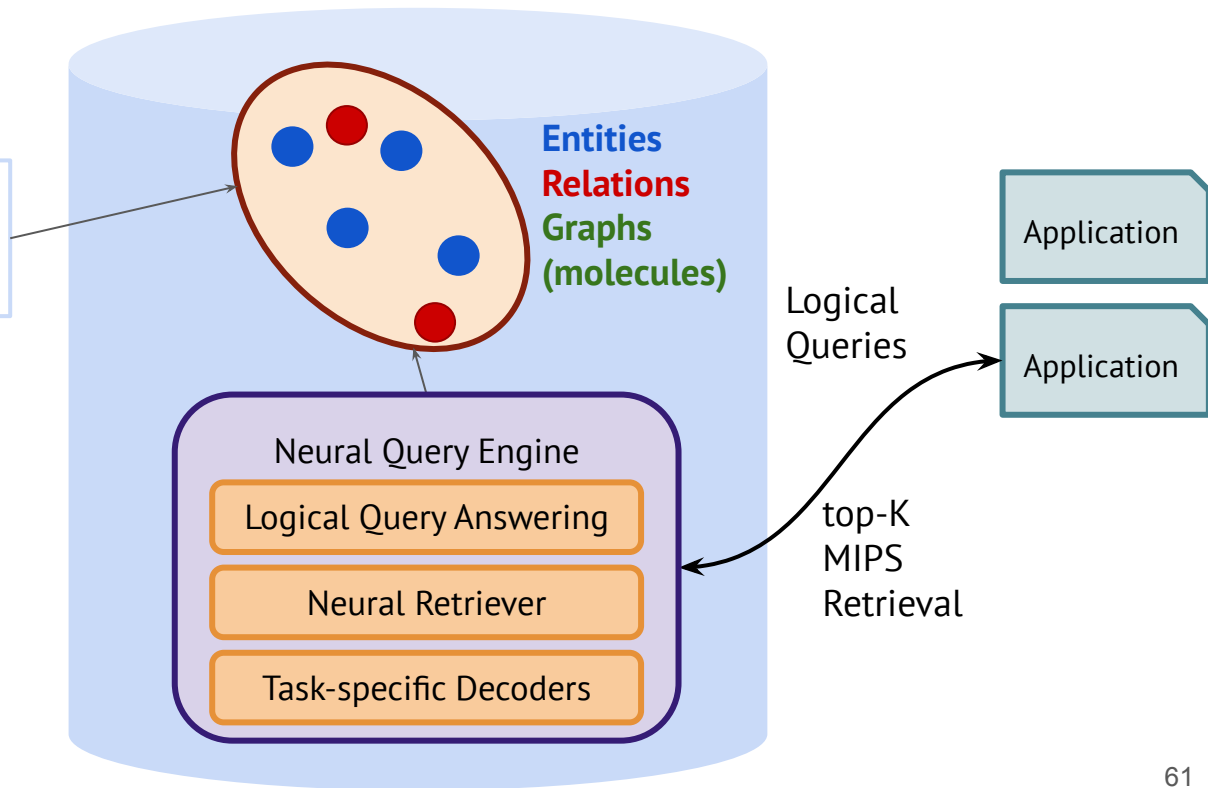
TASK



Future: Neural Graph Databases

(Montreal, location, Quebec)
(Quebec, location, Canada)
(Canada, bordersWith, USA)

- No symbolic storage
- Embedding-based storage
- Inferring Missing Links
- Complex Query Answering
- Updatable





Q&A Time!



@michael_galkin



@mgalkin



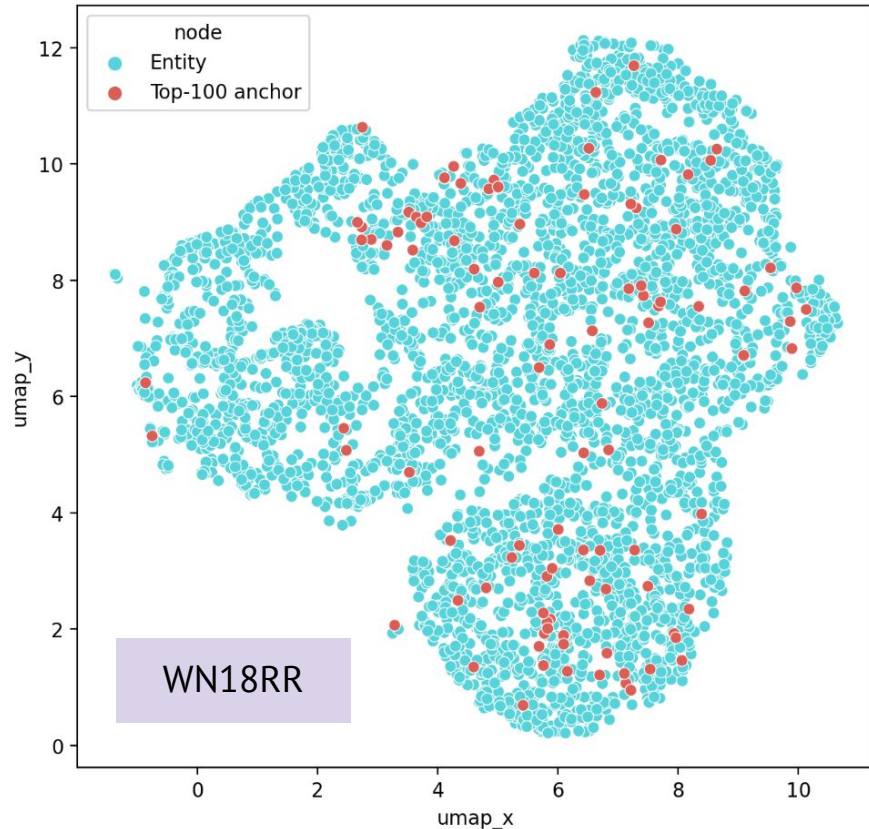
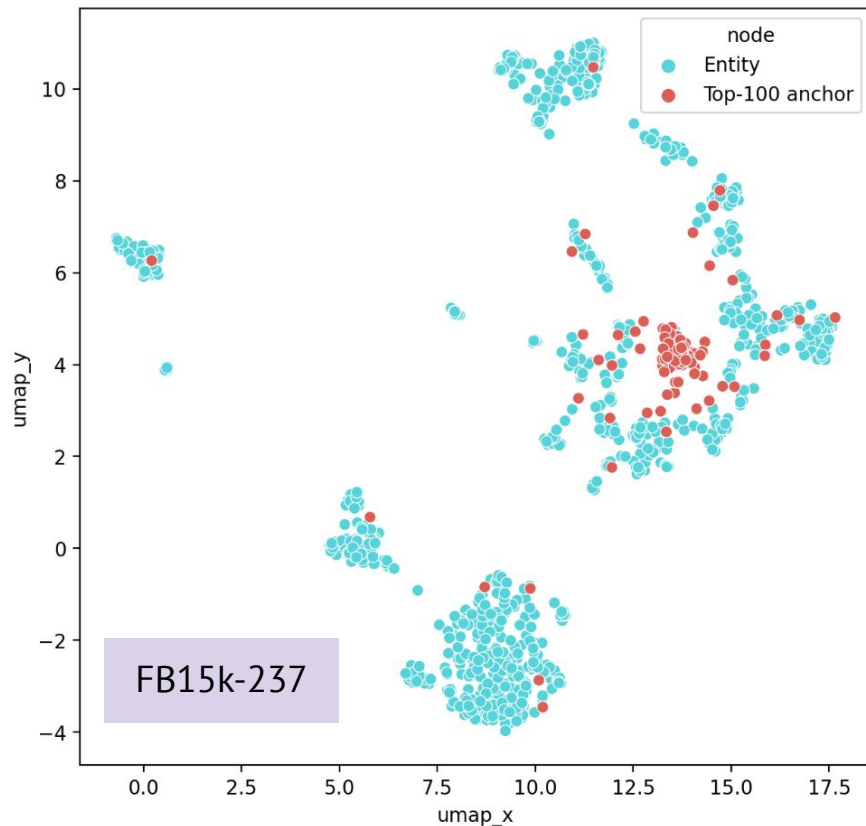
mikhail.galkin@mila.quebec



migalkin.github.io

Backup

Visualizations: Anchors + Entities



Complex Logical Query Answering: Why?

Transductive

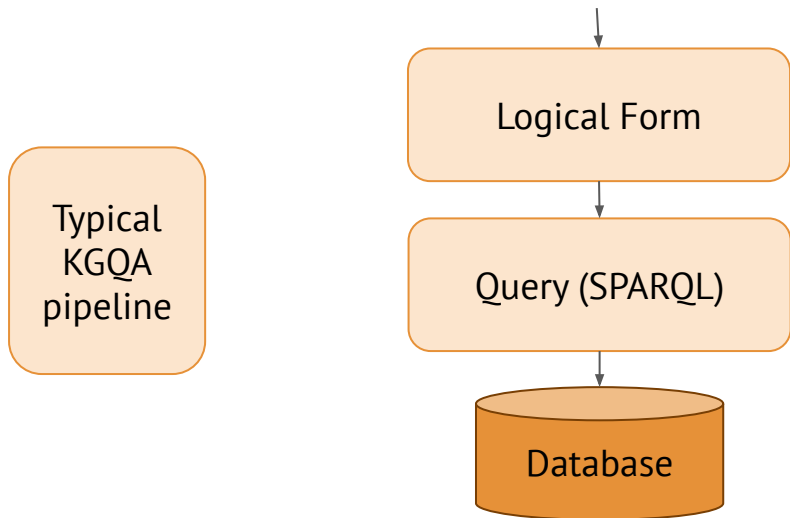
Triples

Inductive

Hyper-relational

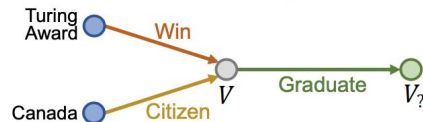
Query Embedding

Where did Canadian citizens with Turing Award graduate?

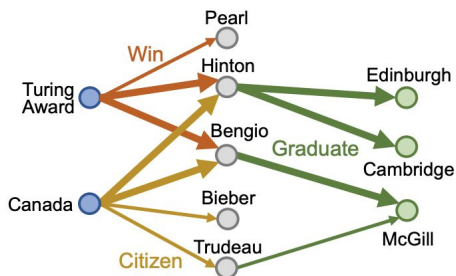


(A) Query q and Its Dependency Graph

$$q = V_1 . \exists V : \text{Win}(\text{TuringAward}, V) \wedge \text{Citizen}(\text{Canada}, V) \wedge \text{Graduate}(V, V_2)$$



(C) Knowledge Graph Space



Complex Logical Query Answering: Why?

Transductive

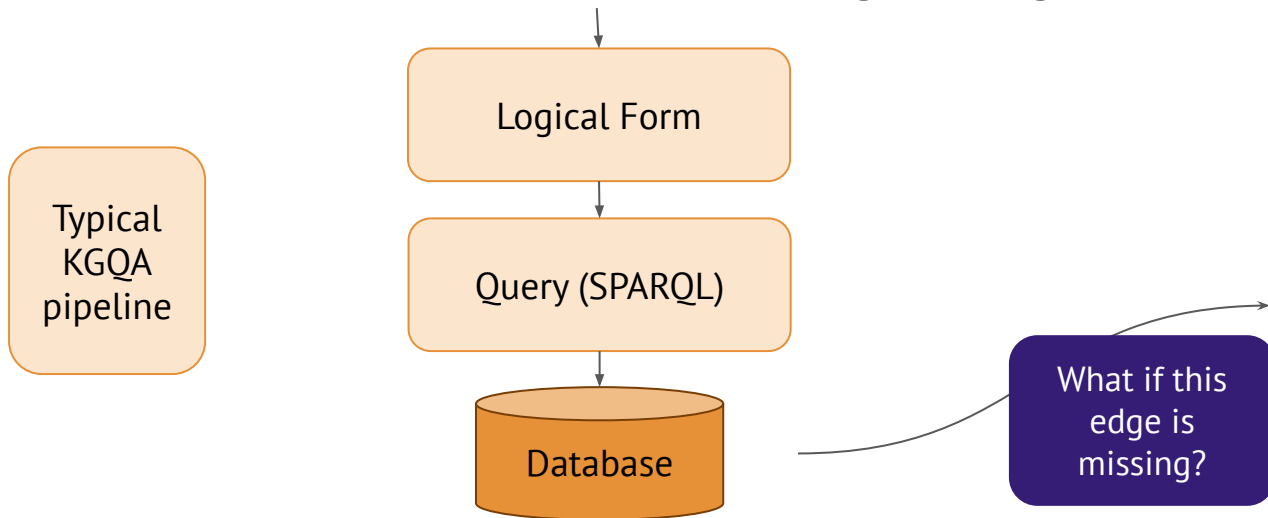
Triples

Inductive

Hyper-relational

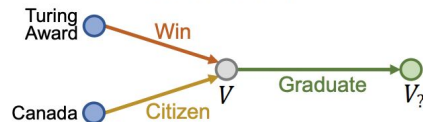
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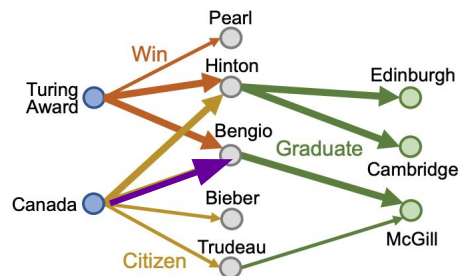


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Complex Logical Query Answering

Transductive

Triples

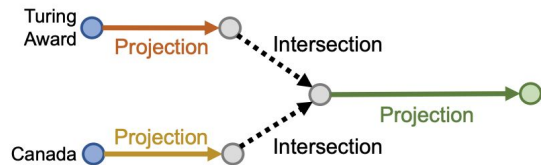
Inductive

Hyper-relational

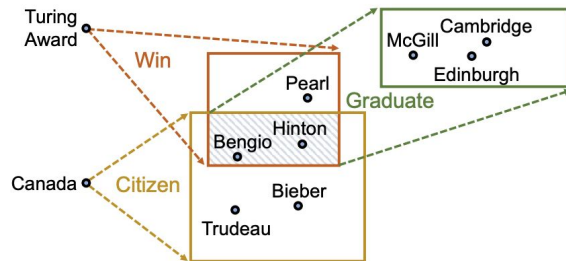
Query Embedding

- Databases assume KGs are **complete**
 - In reality - they are not
- We want to answer FOL queries over **incomplete** graphs with **neural** operators
- Embed a query in a latent space, MIPS decoder for kNN answers

(B) Computation Graph



(D) Vector Space



StarQE: Complex Logical Query Answering on HR KGs

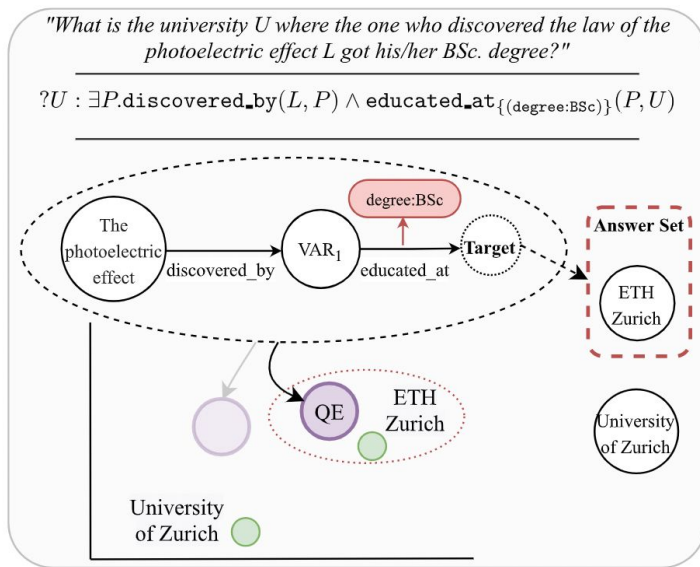
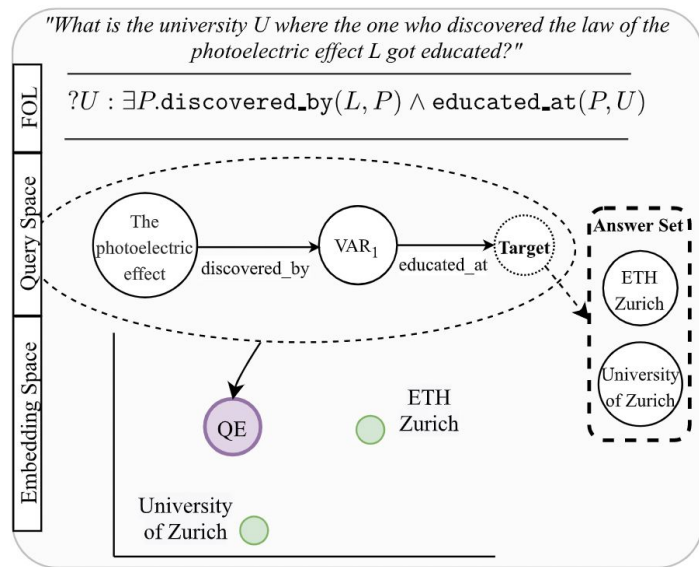
Transductive

Triples

Inductive

Hyper-relational

Query Embedding



1. Extending FOL to HR KGs
2. Qualifiers help A LOT
3. New query types are enabled

★ StarQE for Logical Queries: Summary

- Extend FOL to hyper-relational graphs with qualifiers
- Enabling new query types (eg, joins over qualifier entities)
- Robust to inner representation: RDF* vs reified RDF
- Qualifiers help A LOT in answering complex queries

